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Predicting CO₂ Minimum Miscibility Pressure (MMP) Using Alternating Conditional Expectation (ACE) Algorithm

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Résumé — Prédiction de la pression miscibilité minimum (MMP) du CO₂ en utilisant un algorithme basé sur l'ACE (Alternating Conditional Expectation) — L'injection de gaz miscibles est une des méthodes les plus utilisées pour améliorer la récupération d'hydrocarbures (*Enhanced Oil Recovery*, EOR). En raison du coût important de cette technique, un haut degré de précision est requis pour prédire le processus. Une telle précision comprend les paramètres de dépistage préliminaires pour le déplacement de gaz miscible, la pression minimale de miscibilité (*Minimum Miscibility Pressure*, MMP) et de la disponibilité du gaz.

Toutes les méthodes de mesure du MMP conventionnelles sont des processus consommateurs de temps requérant de ce fait des coûts importants. Par conséquent, afin de répondre aux demandes de réponses rapides du secteur, une approche non paramétrique basée sur l'algorithme ACE (*Alternating Conditional Expectation*) due à Brieman et Friedman [Brieman L., Friedman J.H. (1985) *J. Am. Stat. Assoc.* 80, 391, 580-619], est utilisée dans cette étude pour estimer les MMP. Cet algorithme recherche les transformations optimales d'un ensemble de facteurs prédictifs (ici C₁, C₂, C₃, C₄, C₅, C₆, C₇₊, CO₂, H₂S, N₂, Mw₅₊, Mw₇₊ et T) et d'une réponse (ici MMP) à modéliser qui produisent le maximum de corrélations entre les facteurs et la réponse transformée. Cent treize points de données MMP sont considérés, issus à la fois de la littérature et de travaux expérimentaux. Cinq mesures MMP correspondant à un champ pétrolier koweïtien sont incluses dans les données de test. Le modèle proposé est validé en utilisant une analyse statistique détaillée ; un coefficient de corrélation de 0,956 est obtenu par comparaison avec les corrélations existantes. De même, l'écart type et la moyenne des valeurs d'erreurs absolues sont minimales : respectivement 139 psia (8,55 bar) et 4,68 %. Par conséquent, il s'avère que les résultats sont plus fiables que les corrélations existantes pour l'injection de CO₂ pur pour améliorer la récupération du pétrole. En plus de sa précision, l'approche ACE est plus puissante, rapide et peut gérer un ensemble de données énormes.

Abstract — Predicting CO₂ Minimum Miscibility Pressure (MMP) Using Alternating Conditional Expectation (ACE) Algorithm — Miscible gas injection is one of the most important enhanced oil recovery (EOR) approaches for increasing oil recovery. Due to the massive cost associated with this approach a high degree of accuracy is required for predicting the outcome of the process. Such accuracy includes, the preliminary screening parameters for gas miscible displacement; the “Minimum Miscibility Pressure” (MMP) and the availability of the gas.

All conventional and state-of-art MMP measurement methods are either time consuming or decidedly cost demanding processes. Therefore, in order to address the immediate industry demands a nonparametric approach, Alternating Conditional Expectation (ACE), is used in this study to estimate MMP. This algorithm Breiman and Friedman [Breiman L., Friedman J.H. (1985) J. Am. Stat. Assoc. 80, 391, 580-619] estimates the transformations of a set of predictors (here C_1 , C_2 , C_3 , C_4 , C_5 , C_6 , C_{7+} , CO_2 , H_2S , N_2 , Mw_{5+} , Mw_{7+} and T) and a response (here MMP) that produce the maximum linear effect between these transformed variables. One hundred thirteen MMP data points are considered both from the relevant published literature and the experimental work. Five MMP measurements for Kuwaiti Oil are included as part of the testing data. The proposed model is validated using detailed statistical analysis; a reasonably good value of correlation coefficient 0.956 is obtained as compare to the existing correlations. Similarly, standard deviation and average absolute error values are at the lowest as 139 psia (8.55 bar) and 4.68% respectively. Hence, it reveals that the results are more reliable than the existing correlations for pure CO_2 injection to enhance oil recovery. In addition to its accuracy, the ACE approach is more powerful, quick and can handle a huge data.

NOMENCLATURE

AARE	Average Absolute Relative Error (%)
ARE	Average Relative Error (%)
C_1	Methane mole fraction
C_2	Ethane mole fraction
C_3	Propane mole fraction
C_4	Butane mole fraction
C_5	Pentane mole fraction
C_6	Hexane mole fraction
C_{7+}	Heptane plus mole fraction
CO_2	Carbon dioxide mole fraction
E	Mathematical expectation
ε^2	Regression error
f	Function
HC_{comp}	A group, mole fractions of hydrocarbon composition
H_2S	Hydrogen sulphide mole fraction
Ln	Natural log
Mw_{5+}	Molecular wt of pentane plus
Mw_{7+}	Molecular wt of heptane plus
MMP	Minimum Miscibility Pressure (bar)
NHC_{comp}	A group, mole fractions of non-hydrocarbon composition
N_2	Nitrogen mole fraction
r	Correlation coefficient (%)
RE	Relative Error
r^2	Ratio of data variability
RMSE	Root Mean Square Error
SD	Standard Deviation of the errors
SSE	Sum of Squares of the Errors
SSR	Regression Sum of Squares
SST	Total Sum of Squares

T	Temperature (°C)
Tr	Transform

INTRODUCTION

The injection gases most commonly used for enhanced oil recovery processes are generally not miscible upon first contact with the reservoir fluids that they are displacing. Miscible gas injection into an oil reservoir is among the most widely used enhanced oil recovery techniques and its applications are increasingly evident in oil production worldwide. Two important concepts associated with the description of miscible gas injection processes are the Minimum Miscibility Pressure (MMP) and Minimum Miscibility Enrichment (MME). The MMP has typically been accepted as the pressure at which practical maximum recovery efficiency is observed. In other words, it is the lowest pressure at which gas and oil become miscible at a fixed temperature and the displacement process becomes very efficient (Ayirala and Rao, 2006). It is considered as one of the most important factors in the selection of candidate reservoirs for gas injection at which miscible recovery takes place and it determines the efficiency of oil displacement by gas.

MMP can be measured by employing experimental and non-experimental methodologies. In the industry, there are many experimental techniques available to estimate MMP such as; slim tube (Yellig and Metcalfe, 1980; Huang and Dyer, 1993), rising bubble apparatus (Christiansen and Haines, 1984), multi-contact experiment or mixing-cell experiment (Bryant and Monger, 1988; Menzie and Nielsen, 1963; Turek *et al.*, 1988), pressure-composition diagram (Orr and Jensen, 1984),

vanishing interfacial tension (Gasem *et al.*, 1993), falling drop technique (Zhou and Orr, 1995), vapour density (Harmon and Grigg, 1988) and high pressure visual sapphire cell (Hagen and Kossack, 1986). The non-experimental methods consist of both analytical and numerical approaches. All empirical correlations (Alston *et al.*, 1985; Kuo, 1985; Glaso, 1985; Orr and Silva, 1987) and Equation of State (EOS) also belong to the analytical techniques. In EOS techniques for MMP calculations, the complex multicomponent system is streamlined into its light, medium and heavy ends along with pseudo components. A two phase region is developed and subsequently the critical region identification gives the value of MMP (Yurkiw and Flock, 1994). Non-experimental computational methods are fast and convenient alternatives to otherwise slow and expensive experimental procedures. This research focuses on the analytical aspect of MMP estimation. It introduces a non-parametric model to improve the MMP estimation.

1 MMP DETERMINATION (EXPERIMENTAL TECHNIQUES)

Slim-tube experiment is widely accepted as the industry standard experimental procedure to estimate the MMP. Because of lack of sufficient data points and the small amount of dispersion in the displacements, estimating the slim-tube MMP may be difficult (Johns *et al.*, 2002). *P-X* (Pressure-Composition) diagram, multi contact, core flooding tests and vapour density experiments of injected gas *versus* pressure at low temperatures are generally reliable because they use real fluids and can capture the complex interactions between fluid flow and phase behaviour in a porous medium. These experiments, however, are very slow (usually take number of days or weeks) and relatively expensive to conduct, and thus, after a long laborious endeavour a very few MMP are obtained.

Other experimental MMP methods, such as Rising Bubble Apparatus (RBA) experiments and Vanishing Interfacial Tension (VIT) tests are not reliable and mostly unaccepted because usually they miscarry the reproduction of fluid flow and phase behaviour relations in Condensing-Vaporizing floods (Jessen and Orr, 2008). The RBA test is a fast way to estimate the MMP during vaporizing gas drive injection study. The test involves direct visual observation of the behaviour of a bubble of injection gas as it rises through a column of reservoir oil contained in the RBA cell.

Many investigators (Novosad *et al.*, 1989; Elsharkawy *et al.*, 1992; Huang and Dyer, 1993; Srivastava *et al.*, 1994) had conducted comparison studies between slim

tube and RBA techniques. VIT experimental technique is quite simple in terms of its principles. This method is based on the concept that, at miscibility conditions there is zero Interfacial Tension (IFT) between the two phases. Actually, IFT is measured between the injected gas (solvent) and crude oil at reservoir temperature at varying pressures or enrichment levels of gas phase. Finally, an extrapolation of the plot between IFT and pressure to zero interfacial tension determines MMP (Rao, 1997).

Further to this, a few researchers also investigated the modification of existing experimental methods; Srivastava and Huang (1998) suggested a single bubble injection technique to extend the applicability of RBA to measure MMP for solvent gases exhibiting enriched gas drive behaviour. Kechut *et al.* (1999) proposed Vapour Liquid Equilibrium-Interfacial Tension (VLE-IT) approach that measures the IFT between the injected gas (solvent) and the oil at reservoir temperature and varying pressures using a prototype equipment.

2 MMP DETERMINATION (NON-EXPERIMENTAL TECHNIQUES)

Computational methods provide fast and cheap alternatives to MMP experimental approaches. They are also requisite tools in tuning equations of state to calculate MMP for compositional simulations. Incorporating the MMP in the process of tuning can improve the accuracy of equations of state in gas displacement simulations (Jessen *et al.*, 2004; Yuan *et al.*, 2004). The non-experimental methods for MMP determination are classified into; numerical methods and analytical method.

2.1 Numerical Methods

Assuming appropriate Equation of State (EOS) based fluid phase behaviour characterization is available then MMP can be calculated numerically. EOS's reliability depends on the quality of the data used and the oil composition. It is also demonstrated by Wang and Peck (2000) that among the various available numerical MMP calculation approaches, one-dimensional compositional simulation MMP predictions are very consistent and agree with the slim-tube test data, provided an appropriate fluid phase behaviour characterization is available and numerical dispersion has already been taken into considerations. Dispersion will result in loss of miscibility, as it causes the composition route to enter into the two-phase region. Zick (1986), Stalkup (1987) and Stalkup *et al.* (1990) and others also indicate that

numerical simulation and 1D slim-tube simulation give excellent match to the experimental data. Simulation of a slim-tube is significantly cheaper and faster than running the actual experiments but the phase behaviour of the oil and gas must be well-described by an EOS for a reliable MMP estimation. Slim-tube simulation has a number of drawbacks. First, it is slower and more time-consuming compared to other computational methods. The setup and calibration time is considerably longer than other computational method. In addition, several simulations with varying number of grid blocks are needed for a reliable MMP estimate. Estimating the MMP by 1-D compositional simulation clones the flow in porous media that occurs in slim-tube experiments (Yellig and Metcalfe, 1980).

It is observed that coarse-grid compositional simulations can suffer from numerical dispersion effects, causing the MMP to be in error (Stalkup, 1987; Johns *et al.*, 2002). The effect of dispersion can be reduced but not eliminated. Addition to this, if cells are fine and quite enough so that the numerical dispersion effect can be alleviated then both 2D or 3D compositional simulation models can also be used to calculate MMP very precisely. Though, selecting fine grid blocks will reduce the numerical dispersion detriments but these simulation approaches are generally time-consuming. In addition, when the number of pressure points at which simulations are performed is not large enough to obtain a reasonably well defined recovery curve, the numerically calculated MMP are subjected to the visual interpretation of the recovery curve (Johns *et al.*, 2002).

2.2 Analytical Methods

Analytical methods of estimating MMP use the Method of Characteristics (MOC) (Jessen *et al.*, 1998; Orr *et al.*, 1993; Wang and Orr, 1997). The MOC relies on an equation of state to find a set of key tie lines that govern the oil displacement by gas. The MMP estimation algorithms based on MOC tracks these key tie lines with pressure to find the MMP of the displacement. The MMP occurs at the pressure at which any one of the key tie lines first intersects a critical point (or its length becomes zero). These methods are based on the analytical theory of multi component gas injection processes. Because of their improved speed, analytical methods offer significant promise for developing improved fluid correlations and for use in compositional streamline simulations. Correlations for predicting MMP have been proposed by a number of investigators and are important tools for rapid and accurate MMP calculation. Enich *et al.* (1988) explained that any correlation for the prediction of MMP should be a function of

thermodynamic or physical principles that affect the miscibility of fluids, and should be directly related to the multiple contact miscibility process. For an initial and quick estimate, operators use correlation currently available in the literature. For screening purposes, correlations gave a fair first guess depending on the data used. However, the success of the correlations is usually limited to the composition range in which these correlations were developed. The CO₂ MMP correlations fall into two categories: the pure and impure CO₂; while the other category treats MMP's of other gases. Benham *et al.* (1960) presented empirical curves for predicting MMP for reservoir oils that are displaced by rich gas. Further, he proposed equations that have been derived from his graphical correlations. Cronquist (1978) used the temperature and C₅₊ molecular weight as correlation parameters in addition to the volatile mole percentage (C₁ and N₂). Yellig and Metcalfe (1980) from their experimental study proposed a correlation for predicting the CO₂ MMP's that uses the temperature *T* as the only correlating parameter. Alston *et al.* (1985) presented an empirically derived correlation for estimating the MMP of live oil systems by pure and impure CO₂ streams. MMP has been correlated with temperature, oil C₅₊ molecular weight, volatile oil fraction, intermediate oil fraction and composition of CO₂ streams. Glaso (1985) presented a generalized correlation for predicting the MMP required for multi-contact miscible displacement of reservoir fluids by hydrocarbon, CO₂ or N₂ gas. The equations are derived from graphical correlations given by Benham *et al.* (1960) and give MMP as a function of reservoir temperature, C₇₊ molecular weight, mole per cent methane in the injection gas, and the molecular weight of the intermediates (C₂ through C₆) in the gas. Orr and Jensen (1984) suggested that the vapour pressure curve of CO₂ can be extrapolated and equated with the MMP to estimate the MMP for low temperature reservoirs. However, none of these correlations gives adequate emphasis to oil properties and composition and all fail to accurately predict the miscibility pressure for variety of crude oil types.

3 ANALYTICAL METHODS REGRESSION TECHNIQUES

Regression analysis is a statistical tool for the investigation of relationships between different (independent and dependent) variables. The key benefits of using regression analysis are that it can: indicate if independent variables have a significant relationship with a dependent variable; indicate the relative strength of different independent variables' effects on a dependent variable and make predictions.

3.1 Parametric Regression Analysis

Sometimes, the response (dependent variable) can also depend on a nonlinear function of the explanatory independent variables (predictors). In some cases, the expected form of the non-linear function is known and can be parameterised in terms of functions. For example, polynomial regression consists of performing multiple regressions with different variables in order to find the polynomial coefficients (parameters). These types of regression are known as parametric regression since they are based on models that require the estimation of a finite number of parameters.

3.2 Non-Parametric Regression Analysis

Conventional multiple regression techniques (as explained in earlier section) are very inadequate because they usually necessitate pre-requisite assumptions about the functional forms that relate several (predictor and response) variables. When the relationship between independent and dependent variable is unidentified or indefinite, linear parametric regression is not very promising, rather than misleading. Contrary to parametric-regression, in non-parametric regression there is a considerable flexibility to accept the regression surface. This all leads and inclines to unconventional non-parametric regression (Friedman and Stuetzle, 1981).

It is a form of regression analysis in which the predictor does not take a predetermined form instead is constructed (transformed) according to information derived from the data. Nonparametric regression requires larger sample sizes than regression based on parametric models because the data must supply the model structure as well as the model estimates. Optimum transformations of independent variables facilitate and explain the effect of predictors on response. In nonparametric regression analysis, a number of transformations of variables are available and appraising the optimal transformation of each variable is the most significant (Brieman and Friedman, 1985). This type of regression techniques is based on successive refinements by attempting to define the regression surfaces in an iterative fashion while remaining 'data driven' as opposed to 'model driven'. Non-parametric regression is easy to use and can quickly provide results that reveal the dominant independent variables and relative characteristics of the relationships (Wu *et al.*, 2000). There are many non-parametric tests (Sign test, Wilcoxon Signed-Ranks test, Mann-Whitney U test, Kruskal-Wallis H-test, Jonckheere test, Friedman ANOVA, etc.) being used for analysis purposes. In this particular study, we have

considered an approach called Alternating Conditional Expectations (ACE).

4 ALTERNATING CONDITIONAL EXPECTATIONS (ACE)

Non-parametric regression methods can be broadly classified into those which do not transform the response variable (such as generalised additive models) and those which do (such as Alternating Conditional Expectations, ACE). Moreover, the ACE algorithm can handle variables other than continuous predictors such as categorical (ordered or unordered), integer and indicator variables (Wang and Murphy, 2004).

The present approach to estimate MMP is guided by the view that statistical methods for dealing with data that exhibit strong linear associations are well developed; consequently, many non-standard problems are best addressed by transforming the data to achieve increased linear association. The analysis given here also serves to illustrate the exploratory use of the ACE algorithm to suggest expressions, and the use of r^2 from the ACE transformed variables as a benchmark. In ACE method, the transformed variables exhibit substantially greater linear association than the untransformed variables. One of the principal benefits of the ACE algorithm is that it provides a theoretical standard against which more analytically appealing transformation can be judged (De Veaux, 1989). Another great advantage by using ACE approach lies in its ability to recover the functional forms of variables and to uncover complicated relationships (Wang and Murphy, 2004). It can be applied both in bivariate and multivariate cases and it yields maximum correlations in transformed space (Malallah *et al.*, 2006). A modification of ACE algorithm with graphical (GRACE) interface was later proposed by Xue *et al.* (1997).

5 DEVELOPMENT OF ACE MMP MODEL

This study uses an algorithm (ACE) of Brieman and Friedman (1985) for estimating the transformations of a response and a set of predictor variables in multiple regression problems in enhanced oil recovery.

5.1 Optimal Transformations

In the current study, we have a dataset consisting of a response variable Y (Minimum Miscibility Pressure) and predictor variables $X_1, X_2, X_3, X_4, \dots, X_P$ ($C_1, C_2, C_3, C_4, C_5, C_6, C_{7+}, CO_2, H_2S, N_2, Mw_{5+}, Mw_{7+}$

and T). ACE algorithm defines arbitrary measurable mean-zero transformations as $\theta(Y)$, $\phi_1(X_1)$, $\phi_2(X_2)$, $\phi_3(X_3)$, ..., $\phi_P(X_P)$. The regression error (ε^2) in (under the constraint, $E(\theta^2(Y)) = 1$) the transformation of the dependent and independent variables will be optimal for this regression (Datta-Gupta *et al.*, 1996) if they satisfy the following:

$$\varepsilon^2(\theta^*, \phi_1^*, \phi_2^*, \phi_3^*, \dots, \phi_p^*) = \lim_{(\theta, \phi_1, \dots, \phi_p) \rightarrow \min} \varepsilon^2(\theta, \phi_1, \phi_2, \dots, \phi_p) \quad (1)$$

The correlation coefficient between these optimal transforms variables (with constraints, $E[\theta^2(Y)] = 1$ and $E[\phi_s^2(X)] = 1$) are defined as follows:

$$\rho(\theta, \phi_s) = E[\theta(Y)\phi_s(X)] \quad (2)$$

where:

$$\phi_s(X) = \sum_{l=1}^p \phi_l(X_l)$$

By the same reasoning, transformation $\theta^{**}(Y)$, $\phi_1^{**}(X_1)$, $\phi_2^{**}(X_2)$, $\phi_3^{**}(X_3)$, ..., $\phi_P^{**}(X_P)$ will be optimal for the correlation if:

$$\rho^*(\theta^{**}, \phi_s^{**}) = \lim_{(\theta, \phi_1, \dots, \phi_p) \rightarrow \max} \rho(\theta, \phi_s) \quad (3)$$

5.2 ACE Algorithm

The objective is to minimise the error (ε^2) equation (under the constraint, $E(\theta^2(Y)) = 1$) (Datta-Gupta *et al.*, 1996) as:

$$\varepsilon^2(\theta, \phi) = E\{[\theta(Y) - \phi(X)]^2\} \quad (4)$$

Minimization of error (ε^2) with respect to a predictor $\phi(X)$ for a given $\theta(Y)$ is defined (Datta-Gupta *et al.*, 1996) by:

$$\phi(X) = E[\theta(Y)/X] \quad (5)$$

Similarly, the minimization of error (ε^2) with respect to a response $\theta(Y)$ for a given $\phi(X)$ is defined (Datta-Gupta *et al.*, 1996) by:

$$\theta(Y) = E[\phi(X)/Y]/E\phi(X)/Y \quad (6)$$

Equations (4), (5) and (6) are the foundation of the ACE algorithm.

5.3 ACE Predictions

An important application of ACE technique is for the estimation or prediction of dependent variable (response) y_j^{pre} given independent variables $\{x_{lj}, \dots, x_{lp}\}$. The dependent variable for any data point is calculated (Datta-Gupta *et al.*, 1996) as:

$$Y_j^{pre} = \theta^{*-1} \sum_{l=1}^p \phi_l^*(X_{lj}) \quad (7)$$

The prediction methodology using Equation (8) consists of three steps; the derivation of optimal transformations $\{\theta^*(Y), \phi_1^*(X_1), \phi_2^*(X_2), \dots, \phi_P^*(X_P)\}$ based on observed data, followed by forward transformations of $\{x_{lj}, \dots, x_{pj}\}$ to $\{\phi^*_{x_{lj}}, \dots, \phi^*_{x_{pj}}\}$ and finally a backward transformation (Datta-Gupta *et al.*, 1996) as:

$$\theta^{*-1} \sum_{l=1}^p \phi_l^*(X_{lj}) \quad (8)$$

6 INVESTIGATING THE FACTORS AFFECTING MMP

MMP depends upon the composition of the injected gas, the reservoir temperature, and the characteristics of the in place fluid. On the other hand, this pressure of miscibility is independent of the nature of the porous media or of the velocity of displacement. Generally, MMP increases steadily with increasing temperature, and oils with higher density and molecular weight have a higher MMP. It has been reported that even small impurities, can significantly affect the miscibility pressure (Glaso, 1985). Alston *et al.*, (1985) documented the fact that the achievement of miscibility is strongly related to reservoir temperature and oil composition, particularly C_5+ molecular weight. Holm and Josendal (1974) found that MMP was only affected by the type of hydrocarbons present in the range C_5 to C_{30} fractions of the crude oil. Yellig and Metcalfe (1980) found little significance of C_7+ properties of the oil on the CO_2 MMP. Alston *et al.* (1985) have shown that the reservoir oil volatile and intermediate fractions can significantly affect the MMP when their ratios depart from unity. This also explained the effects of both solution gas (live oil systems) and impurity of CO_2 sources (Alston *et al.*, 1985).

Johnson and Pollin (1981) presented an empirical correlation which predicted the MMP for a wide variety of live oils and stock oils with both pure and diluted CO_2 . This correlation, requiring only the oil gravity, molecular weight, reservoir temperature and injection gas composition, showed substantially better agreement with experiment tests. Many correlations relating the MMP to

the physical properties of the oil and the displacing gas have been proposed to facilitate screening procedures and to gain insight into the miscible displacement process (Alston *et al.*, 1985; Orr and Silva, 1987; Rathmell *et al.*, 1971).

To study the effect of these parameters on tested data, several sensitivity analyses were conducted. Figure 1 shows the relationship between the independent variables and MMP for the data used in this study. The correlation coefficient of each independent variable is shown. It is clear that the temperature is the most dominant factor. The parameters C₁, C₂, C₅ and Mw₅₊ have the same proportional effect with C₅ and Mw₅₊ having insignificant roles. On the other side, intermediate components (C₃, C₄) and heavy ends (C₆, C₇₊) are showing inverse effects. Addition to this, all the non-hydrocarbon gases (CO₂, N₂ and H₂S) show an inverse relationship with a considerable role.

7 PROPOSED CO₂ MMP MODEL FORMULATION

As discussed earlier, MMP is a function of temperature, crude oil composition and composition of the solvent.

To understand the *in situ* crude oil composition impact on MMP, the functional form of MMP Model is:

$$MMP = f(HC_{comp}, NHC_{comp}, T, Mw_{5+}, Mw_{7+}) \quad (9)$$

- HC_{comp} Mole fraction of hydrocarbons (C₁, C₂, . . . , C₇₊)
- NHC_{comp} Mole fraction of non-hydrocarbons (H₂S, CO₂, N₂)
- T Temperature
- Mw₅₊ Molecular weight of pentane Plus
- Mw₇₊ Molecular weight of heptane Plus

7.1 Data Distribution

The data set used in this study consisted of 113 MMP measurements (pure CO₂) taken from worldwide gas injection projects published in the literature. The ranges of independent variables and MMP used for this study are shown in Table 1. The collected data cover a wide range of API gravities (13 to 48 °API) and reservoir temperatures (21.67°C / 71°F to 129.44°C / 265°F). The data were divided into two sets. The training set consisted of 96 MMP measurements and a testing set of 17, which

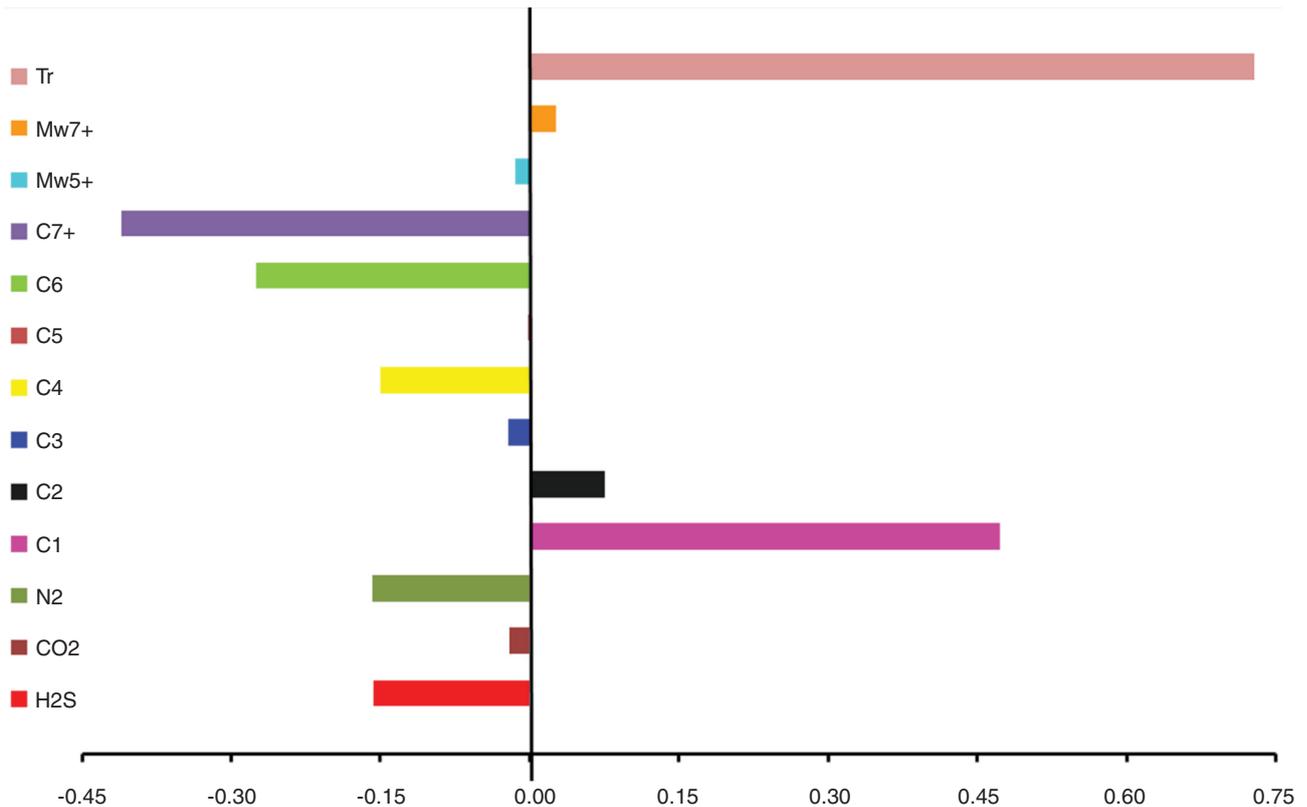


Figure 1
CO₂ MMP sensitivity analysis.

TABLE 1
Data range used for input variables

	H ₂ S	CO ₂	N ₂	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆	C ₇₊	MC ₅ +	MC ₇ +	T (°C) (F)	MMP (bar) (psig)
Max	17.56	24.00	16.44	76.43	23.16	18.40	11.23	9.49	16.00	73.61	262	286	129 (265)	256 (3 705)
Min	0.001	0.001	0.001	2.56	0.01	0.89	0.28	0.25	0.79	5.70	132	151	22 (71)	76 (1 101)

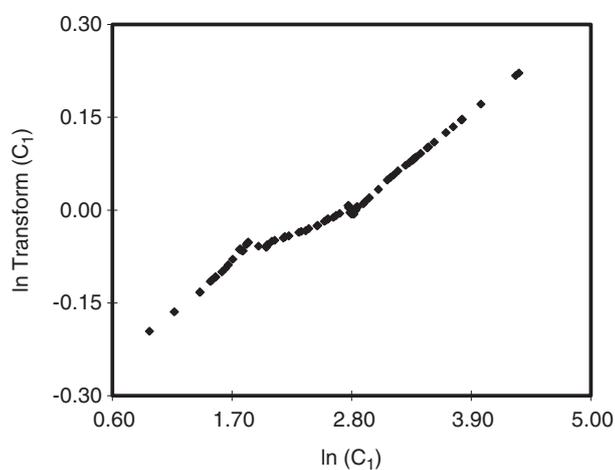


Figure 2
C₁ ACE optimal transform.

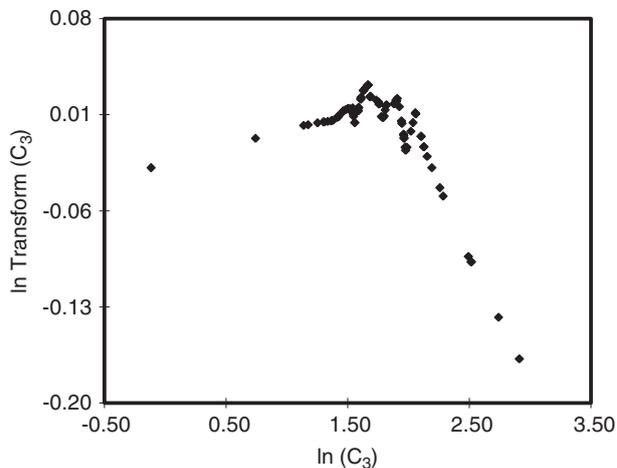


Figure 4
C₃ ACE optimal transform.

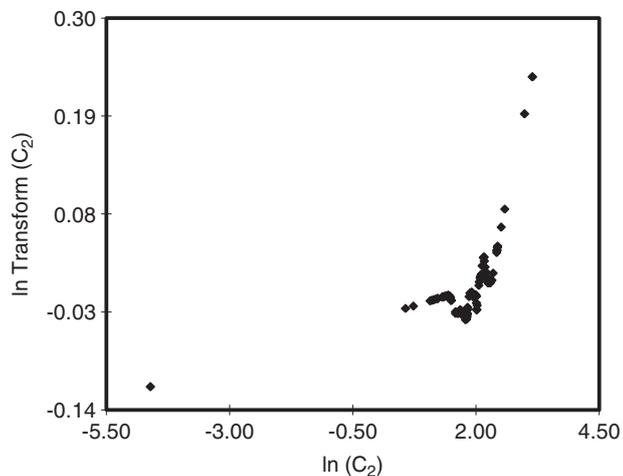


Figure 3
C₂ ACE optimal transform.

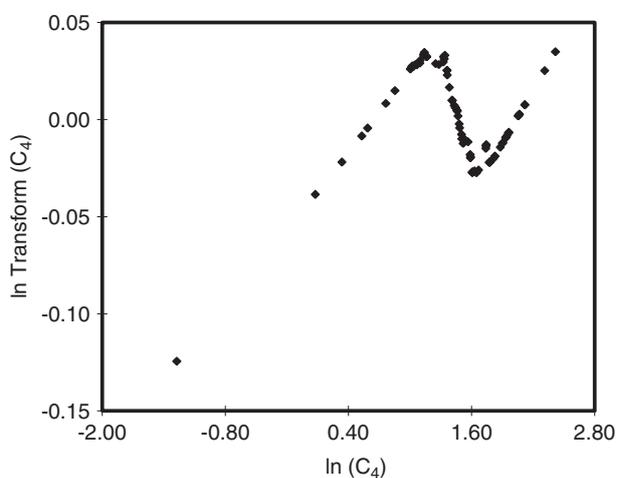


Figure 5
C₄ ACE optimal transform.

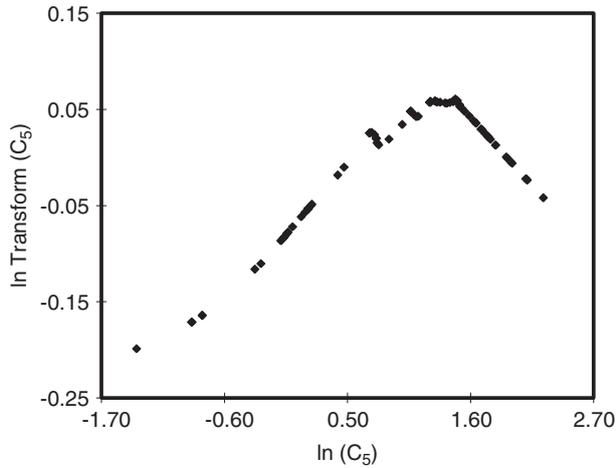


Figure 6
C₅ ACE optimal transform.

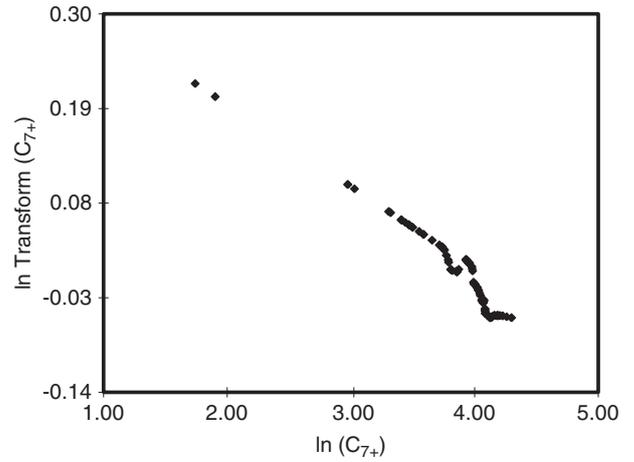


Figure 8
C₇₊ ACE optimal transform.

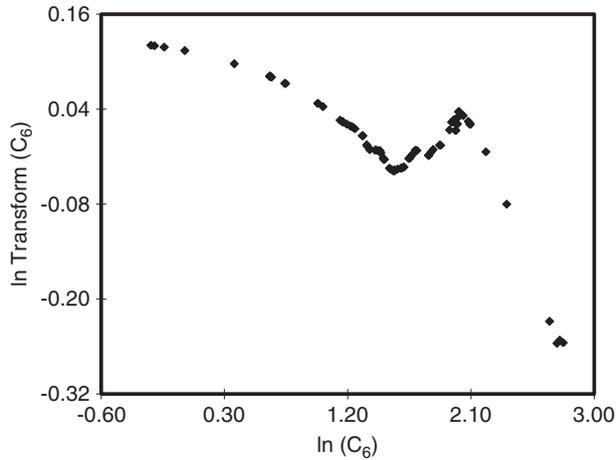


Figure 7
C₆ ACE optimal transform.

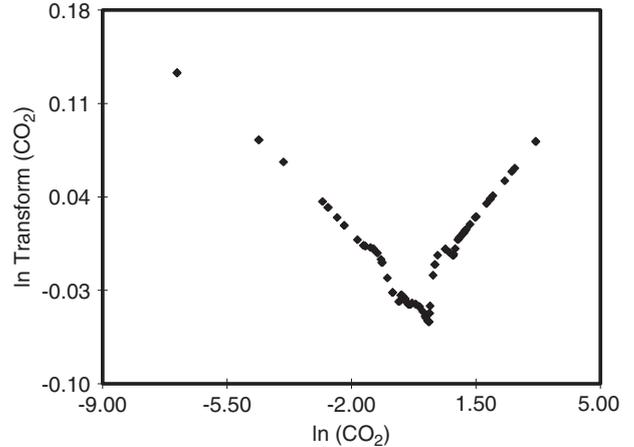


Figure 9
CO₂ ACE optimal transform.

were randomly selected from the total set of data. Out of 17 testing set, 5 were taken from MMP measurements of Kuwait oil fields studied in Kuwait University PVT Lab. Both, the detailed compositional analyses and Minimum Miscibility Pressure measurements were experimentally determined.

7.2 Optimal ACE Regression

The proposed ACE algorithm provides a nonparametric optimization of the dependent (MMP) and independent variables (HC_{comp} , NHC_{comp} , T , Mw_{5+} , Mw_{7+}), it does not provide a computational model for these variables. However, the optimal data transforms can be fitted by

simple polynomials that can be used to predict the dependent variable. The default polynomial is of degree two but for any improvement the degree can be increased. After testing all possible combinations of the independent and dependent variables, the complete suite of fitting polynomials is listed in appendix. Coefficients of this fitted polynomial will be incorporated in the final SUM equation to estimate MMP.

In the proposed nonparametric ACE model for MMP estimation, there are 13 predictors. The sum of all these optimal transformed independent variables is:

$$SUM = \sum_{i=1}^{13} p_i \quad (10)$$

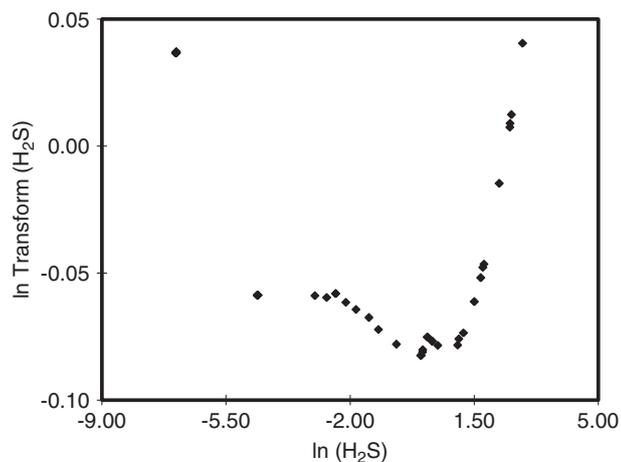


Figure 10
H₂S ACE optimal transform.

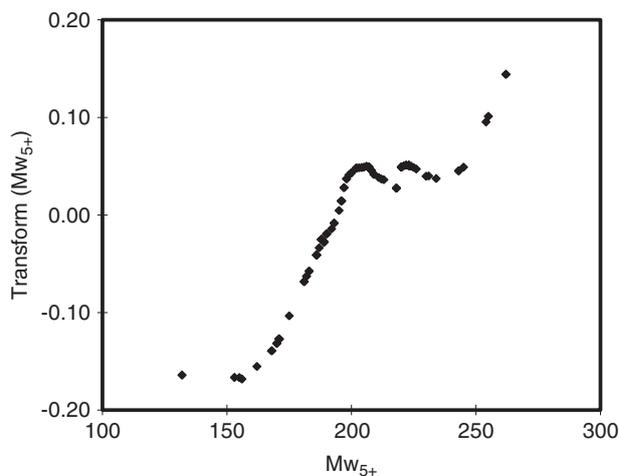


Figure 12
Mw₅₊ ACE optimal transform.

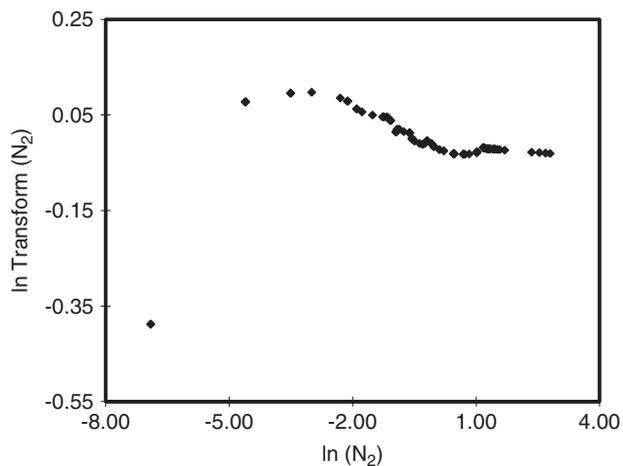


Figure 11
N₂ ACE optimal transform.

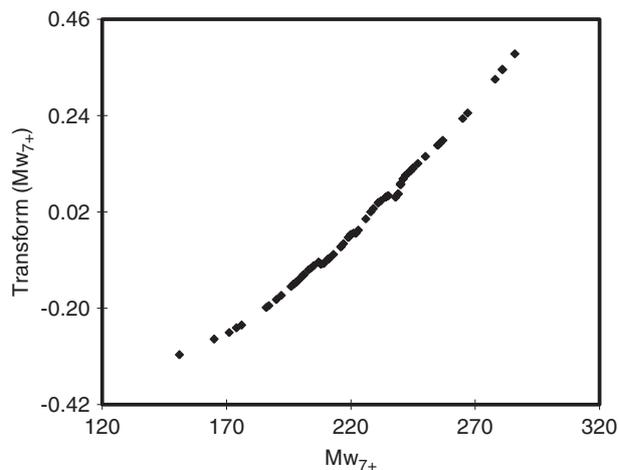


Figure 13
Mw₇₊ ACE optimal transform.

ACE predicted/calculated MMP will be:

$$\text{MMP} = p_0^{-1}(\text{SUM}) \quad (11)$$

A thorough investigation and detailed scrutiny of several scenarios for different transforms of predicting variables was exercised. This yields the best combination, that has the highest correlation coefficient ($r^2 = 0.956$), the lowest Average Absolute Relative Error (AARE = 4.68%), the lowest Average Relative Error (ARE = -0.66%), and the lowest Standard Deviation (SD = 139) is;

$$\text{MMP} = a_2 \text{SUM}^2 + a_1 \text{SUM} + a_0 \quad (12)$$

8 RESULTS AND DISCUSSION

8.1 Optimal Transforms Predictions

The nonparametric independent variables proposed in this study (HC_{comp} , NHC_{comp} , T , Mw_{5+} , Mw_{7+}) were investigated thoroughly to find their optimal transforms using ACE. Similarly, the nonparametric dependent variable (MMP) was investigated.

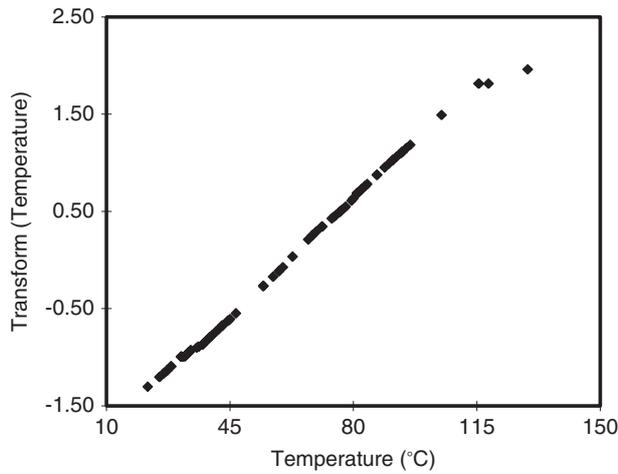


Figure 14
Temperature ACE optimal transform.

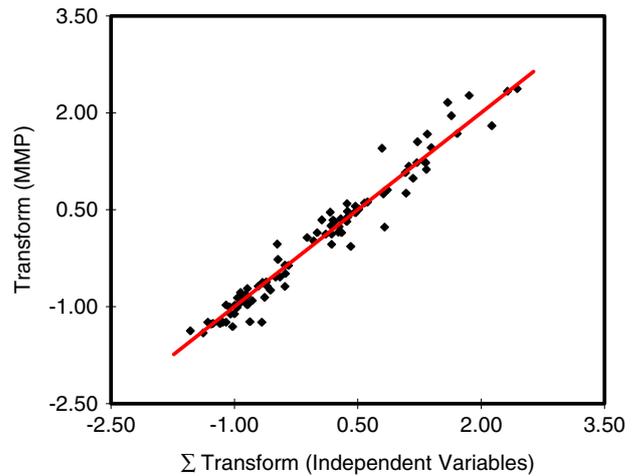


Figure 16
MMP_Tr versus sum of optimal transforms (independent variables) (ACE optimal regression).

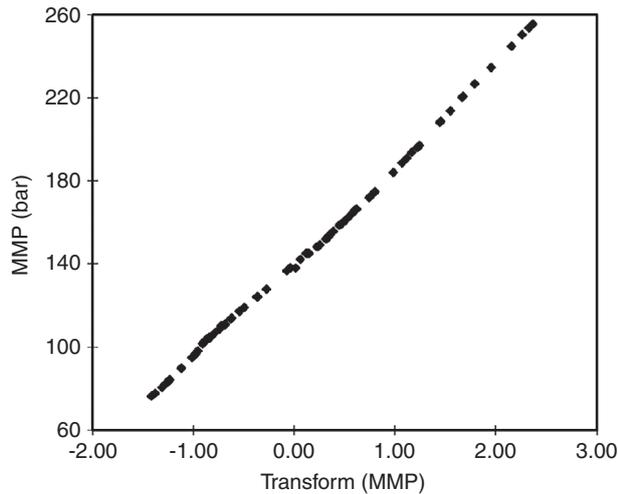


Figure 15
MMP (dependent variable) ACE optimal transform.

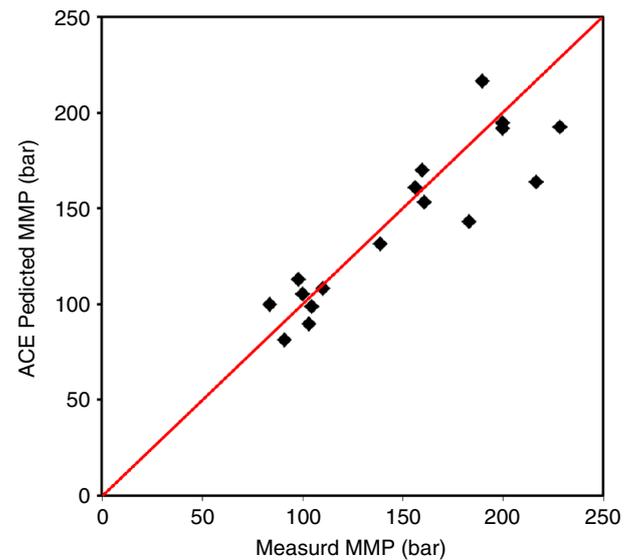


Figure 17
Cross-plot of CO₂ MMP testing data.

In case of hydrocarbon transforms, all mole fractions are found more optimal to their respective transforms in the logarithmic space instead of real space. These optimal transforms defined by ACE are explained in Figures 2-8. The default polynomial interpreted by Graphical Representation of ACE (GRACE) is of degree 2 like in case of C₁ and C₇₊, however there are some higher degrees polynomials; C₂ and C₄ with degree 6, C₆ with degree 5, and C₃ and C₅ with degree 4. Each ACE defined transform shows a specific pattern.

The non-hydrocarbon transforms of all mole fractions are found more optimal to their respective transforms in the logarithmic space instead of real space. These optimal transforms defined by ACE are explained in Figures 9-11. In this group, we have higher degrees of fitting polynomials; CO₂ with degree 6 and H₂S and N₂ both with degree 4.

The real space optimal transforms for both plus fraction molecular weights (Mw₅₊ and Mw₇₊) and temperature transforms are more optimal to their

TABLE 2
Statistical analysis for MMP (training data)

Model	AARE (%)	ARE (%)	SD	SSE	r (correlation)	r ² (determination)	RMSE
ACE	4.68	-0.66	638.11	1.86E+06	0.9783	0.9571	132.21
Alston	14.40	-1.94	926.33	2.59E+07	0.8369	0.7004	507.03
Cronsqt	12.97	-11.26	928.92	2.25E+07	0.9159	0.8389	372.87
Yell-Metcalf	15.58	14.89	635.18	1.47E+07	0.9187	0.8440	250.87
Orr-Jensen	15.36	2.63	932.18	1.50E+07	0.9291	0.8632	344.75
Glaso	7.99	5.39	648.83	5.91E+06	0.9437	0.8906	214.64

TABLE 3
Statistical analysis for MMP (testing data)

Model	AARE (%)	ARE (%)	SD	SSE	r (correlation)	r ² (determination)	RMSE
ACE	10.22	2.84	509.09	1.60E+06	0.9046	0.8183	217.01
Alston	22.65	-0.03	791.11	8.18E+06	0.7926	0.6282	482.37
Cronsqt	15.56	-13.75	735.77	5.12E+06	0.9265	0.8584	276.87
Yell-Metcalf	15.93	15.93	501.26	3.23E+06	0.9178	0.8424	199.02
Orr-Jensen	15.06	3.84	828.79	3.65E+06	0.8842	0.7818	387.13
Glaso	8.54	2.92	481.15	1.89E+06	0.9132	0.8339	196.08

respective transforms in the real space. These optimal transforms defined by ACE are explained in Figures 12-14. Both Mw_{7+} and T transforms are fitted at default polynomials of degree 2, whereas Mw_{5+} transform is fitted at degree 4.

Finally, the real space of dependent variable (MMP) optimal transform is shown in Figure 15. This transform has a fitting polynomial of default degree 2. Coefficients of this fit polynomial will be incorporated in the final SUM equation to estimate MMP.

All transformed independent variables (predictors) are found numerically, finally they are added up and correlate with the transform of MMP. It is shown in Figure 16 that an excellent correlation is obtained and this match is modelled by Equation (13), as shown below. The correlation coefficient for this optimized regression is 0.98014. This proves the power of ACE algorithm.

$$\text{MMP} = 25.923 \text{ SUM} \times Tr^2 + 651.360 \text{ SUM} \times Tr + 2009.7 \quad (13)$$

8.2 Comparison of Proposed Model with Existing Correlations

A detailed comparison study was conducted between the proposed CO₂ MMP model and the existing published and well acknowledged CO₂ MMP correlations. Table 2 presents the statistical analysis between the results predicted by ACE model and that by published correlations. It is inferred from the analysis that our proposed method shows high accuracy as compared to other correlations.

All statistical parameters; AARE, ARE, SD, SSE, RMSE, r and r^2 are depicting good commitment. Therefore, it is inferred from the statistical analysis that ACE is a powerful tool and shows high accuracy as compared to other available predicting correlations.

8.3 Validation of Proposed ACE Model for CO₂ MMP

To check the validity/credibility of ACE model and to check its predictive capability for MMP, all the derived polynomials of variables (both predictors and response) were examined using testing data of 17 points. Five of these are the experimental measurements made for Kuwaiti oil fields.

Hence, a good match between the experimentally measured and ACE estimated values for MMP is observed in Figure 17. This proves the validity of our proposed ACE model.

Once again a detailed comparison study was conducted for these 17 data points between the proposed CO₂ MMP model and the existing published and well acknowledged CO₂ MMP correlations. A detailed statistical analysis with different statistical parameters is explained in Table 3.

Once again, statistical parameters; SD and SSE are proving good commitment as compared to the other predicting methods. Hence, the overall performance of our proposed method for predicting CO₂ MMP is better and acceptable.

CONCLUSIONS

A nonparametric model to predict MMP is developed based on 96 measurements. Our proposed method using ACE model is shown to be more accurate than the existing conventional regression correlations. This model is able to predict MMP for pure CO₂ as a function of temperature and composition (all possible factors affecting MMP). The model has certain advantages:

It provides a straightforward method for identifying functional relationships between dependent and independent variables and solves the general problem of establishing the linearity assumption that is generally required in regression analysis. Examination of these results can give the data analyst insight into the relationships between these variables, and further suggest if transformations are required.

This non-parametric approach is exclusively data driven and does not assume *a priori* functional form as parametric models do. This feature gives a model more flexible for self-adjustment in order to adjust various ranges of data. The proposed technique can easily be incorporated into integrated flow modelling, production optimization and reservoir simulation softwares.

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APPENDIX

Variable	Optimal transform polynomials
C ₁	$\ln_C_{1_Tr} = 1.3912E-02x^2 + 3.2202E-02x - 1.8611E-01$
C ₂	$\ln_C_{2_Tr} = 5.5795E-04x^6 - 6.7876E-03x^5 + 1.0678E-02x^4 + 1.2248E-01x^3 - 4.5724E-01x^2 + 5.2793E-01x - 2.0734E-01$
C ₃	$\ln_C_{3_Tr} = 1.6381E-02x^4 - 1.3052E-01x^3 + 2.6607E-01x^2 - 1.2019E-01x - 4.3356E-02$
C ₄	$\ln_C_{4_Tr} = -5.1912E-02x^6 + 2.7982E-01x^5 - 3.7272E-01x^4 - 2.4664E-01x^3 + 7.1682E-01x^2 - 2.7774E-01x - 1.3485E-02$
C ₅	$\ln_C_{5_Tr} = -1.9674E-04x^4 - 2.7402E-02x^3 + 1.8143E-03x^2 + 1.4054E-01x - 7.2174E-02$
C ₆	$\ln_C_{6_Tr} = -5.4686E-02x^5 + 2.6628E-01x^4 - 3.6496E-01x^3 + 7.6998E-02x^2 - 8.3604E-04x + 1.1194E-01$
C ₇₊	$\ln_C_{7+Tr} = -2.4459E-02x^2 + 3.8895E-02x + 2.1657E-01$
CO ₂	$\ln_CO_{2_Tr} = 9.4029E-06x^6 - 3.2025E-05x^5 - 1.0969E-03x^4 - 8.1253E-04x^3 + 2.0166E-02x^2 + 1.3278E-02x - 3.2906E-02$
H ₂ S	$\ln_H_{2S_Tr} = 3.0640E-04x^4 + 2.7747E-03x^3 + 6.3899E-03x^2 - 4.0849E-03x - 7.9337E-02$
N ₂	$\ln_N_{2_Tr} = -3.3252E-04x^4 + 6.1601E-04x^3 + 8.4462E-03x^2 - 2.5561E-02x - 8.4381E-03$
M _{w5+}	$M_{w5+Tr} = 1.9056E-08x^4 - 1.5287E-05x^3 + 4.5136E-03x^2 - 5.7789E-01x + 2.6867E+01$
M _{w7+}	$M_{w7+Tr} = 1.5634E-05x^2 - 1.5754E-03x - 4.4180E-01$
Temp	$Temp_Tr = -7.9731E-06x^2 + 2.1085E-02x - 2.8306E+00$
MMP	$MMP = 2.5923E+01 \text{ SumTr2} + 6.5136E+02 \text{ SumTr} + 2.0097E+03$