IMPACT OF MODEL ERROR ON THE MEASUREMENT OF FLOW PROPERTIES NEEDED TO DESCRIBE FLOW THROUGH POROUS MEDIA

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LA RÉPERCUSSION DE L'ERREUR DE MODÈLE SUR LA MESURE DES PROPRIÉTÉS D'UN DÉBIT NÉCESSAIRES POUR DÉCRIRE CE DERNIER À TRAVERS UN MILIEU POREUX

Les milieux indirects sont souvent utilisés pour déterminer les propriétés fondamentales d'un débit nécessaires pour décrire ce dernier à travers des milieux poreux. Par conséquent, si un ou plusieurs postulats sous-jacents dans la description mathématique de ces méthodes indirectes ne sont pas valables, une erreur significative de modèle peut s'introduire dans la valeur mesurée d'une propriété du débit. En particulier, cette étude montre que les courbes de mobilité réelles qui incluent l'effet d'accouplement visqueux entre les phases fluides diffèrent d'une manière significative de celles excluant cet accouplement. D'autre part, l'on montre qu'une différence significative existe entre les mobilités conventionnelles effectives correspondant à un débit en aval à l'état stable, un débit en amont à l'état stable et une imbibition en amont pure. Il semble donc que les mobilités effectives traditionnelles ne sont pas de vrais paramètres mais qu'elles sont infiniment non uniques. De même, il est prouvé que tandis que l'omission des forces hydrodynamiques introduit une valeur peu significative d'erreur de modèle dans la courbe différentielle des pressions du débit en aval en milieux poreux non consolidés, cette omission introduit une valeur importante d'erreur de modèle dans la courbe différentielle de pressions du débit en amont dans ces milieux poreux. En outre, cette omission complique l'explication selon laquelle les gradients de pression correspondant au débit en amont à l'état stable sont de signe opposé. Il est démontré également que l'utilisation non appropriée de la condition de limite d'entrée peut introduire une erreur de modèle significative dans l'étude. Cela est dû au fait suivant : si l'on utilise un noyau court axé sur l'une des méthodes d'état non stable pour déterminer la mobilité effective, de nombreux volumes de pore d'injection peuvent s'avérer nécessaires avant que la saturation d'entrée ne monte à sa valeur maximale. Cela est en contradiction avec l'hypothèse ordinaire selon laquelle la saturation d'entrée monte immédiatement à sa valeur maximale. Finalement, il est signalé que, du fait des écarts de régime et d'échelle de débit, les mobilités effectives mesurées en laboratoire peuvent ne pas être appropriées pour leur inclusion dans la base de données pour une simulation à l'échelle d'un réservoir.

IMPACT OF MODEL ERROR ON THE MEASUREMENT OF FLOW PROPERTIES NEEDED TO DESCRIBE FLOW THROUGH POROUS MEDIA

Indirect methods are commonly employed to determine the fundamental flow properties needed to describe flow through porous media. Consequently, if one or more of the postulates underlying the mathematical description of such indirect methods is invalid, significant model error can be introduced into the measured

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value of the flow property. In particular, this study shows that effective mobility curves that include the effect of viscous coupling between fluid phases differ significantly from those that exclude such coupling. Moreover, it is shown that the conventional effective mobilities that pertain to steady-state, cocurrent flow, steady-state, countercurrent flow and pure countercurrent imbibition differ significantly. Thus, it appears that traditional effective mobilities are not true parameters; rather, they are infinitely nonunique. In addition, it is shown that, while neglect of hydrodynamic forces introduces a small amount of model error into the pressure difference curve for cocurrent flow in unconsolidated porous media, such neglect introduces a large amount of model error into the pressure difference curve for countercurrent flow in such porous media. Moreover, such neglect makes it difficult to explain why the pressure gradients that pertain to steady-state, countercurrent flow are opposite in sign. It is shown also that improper handling of the inlet boundary condition can introduce significant model error into the analysis. This is because, if a short core is used with one of the unsteady-state methods for determining effective mobility, it may take many pore volumes of injection before the inlet saturation rises to its maximal value, which is in contradiction with the usual assumption that the inlet saturation rises immediately to its maximal value. Finally, it is pointed out that, because of differences in flow regime and scale, the effective mobilities measured in the laboratory may not be appropriate for inclusion in the data base for a reservoir-scale simulation.

LA REPERCUSIÓN DEL ERROR DE MODELO SOBRE LA MEDICIÓN DE LAS PROPIEDADES DE UN FLUJO NECESARIAS PARA DESCRIBIR ESTE ÚLTIMO A TRAVÉS DE UN MEDIO POROSO

Se utilizan frecuentemente los medios indirectos para determinar las propiedades fundamentales de un flujo necesarias para la descripción de este último a través de un medio poroso. Por consiguiente, en caso de que varios postulados subyacentes en la descripción matemática de estos métodos indirectos no sean valederos, puede introducirse un error significativo de modelo en el valor medido de una propiedad de flujo. En particular, este estudio tiene por propósito demostrar que las curvas de movilidad reales que influyen en el efecto de acoplamiento viscoso entre las fases fluidas difieren de forma significativa de aquellas que excluyen tal acoplamiento. Por otra parte, se demuestra que existe una diferencia significativa entre las movilidades convencionales efectivas que corresponden a un flujo en situación posterior al estado estable, un flujo en posición anterior al estado estable y una imbibición anterior pura. Por consiguiente, parece que las movilidades efectivas tradicionales no corresponden a verdaderos parámetros, pero, que en cambio, son infinitamente no únicas. Del mismo modo, se demuestra que mientras que la omisión de las fuerzas hidrodinámicas introduce un valor poco significativo de error de modelo en la curva diferencial de las presiones del flujo en la parte posterior en medio poroso no consolidado, esta omisión introduce un valor importante de error de modelo en la curva diferencial de presiones de flujo en la parte anterior en estos medios porosos. Además, esta omisión complica la explicación por la cual los gradientes de presión correspondientes al flujo anterior en estado estable son de signo opuesto. También se demuestra que la utilización inadecuada de la condición de límite de entrada puede introducir un error de modelo significativo en el estudio. Ello se debe al hecho siguiente : si se utiliza un núcleo corto centrado sobre uno de los métodos de estado inestable para determinar la movilidad efectiva, pueden ser necesarios numerosos volúmenes de poro de inyección antes de que la saturación de entrada ascienda a su valor máximo. Todo ello se encuentra en contradicción con la hipótesis ordinaria según la cual la saturación de entrada asciende inmediatamente hasta su valor máximo. Finalmente, se indica que, debido a las diferencias de régimen y de escala de flujo, las movilidades efectivas medidas en laboratorio pueden no resultar adecuadas para su inclusión en la base de datos para una simulación a escala de un yacimiento.

INTRODUCTION

To undertake a waterflood prediction, information concerning flow properties, such as effective mobility and capillary pressure, is needed. Because of the complexity of flow through porous media, it is usually necessary to acquire such information experimentally. Moreover, depending upon the parameter, and, in some cases, on the technique used to measure it, it may not be possible to determine a given parameter directly; rather, indirect methods may have to be used to acquire the information necessary to determine it. When indirect methods are employed, one must keep in mind that such methods are no better than the theory upon which they are based. That is to say, if the assumptions underlying the theory upon which the indirect method is based are not well met, or if they are invalid, model error can enter the analysis.

The effective mobility characteristics of a reservoir rock may be determined by either steady-state or unsteady-state methods. The steady-state methods are more time consuming than the unsteady-state methods. Moreover, the flow taking place during the course of an unsteady-state experiment is more representative of that taking place in the field than is the case in a steady-state experiment. Consequently, the unsteady-state methods are usually preferred. Unsteady-state methods are based on Buckley-Leverett theory (Buckley and Leverett, 1942). Consequently, it is important that the fluids used in effective mobility experiments be incompressible and immiscible, and that the porous media used for these experiments be homogeneous and isotropic.

Moreover, the pressure and saturation must be uniform at each cross-section along the length of the core (Collins, 1961). Also, if a Lagrangian formulation is to be used, it is necessary that saturation decreases monotonically along the length of the core (Bentsen and Saeedi, 1981).

Finally, it is important that the inlet boundary condition be handled properly (Bentsen, 1978; Shen *et al.*, 1994). The violation of any of these assumptions can introduce significant model error into the measured effective mobility curves (Bentsen and Sarma, 1989).

In the conventional formulation of the immiscible, two-phase flow problem (Muskat, 1982), it has been usual to assume that the flux of each phase is proportional to only one driving force, the potential gradient acting across the phase. Such an approach neglects the possibility that momentum transfer between the two flowing phases may act also as a driving force. If such is the case, then the total flow of a given phase, say phase 1, must be the sum of the flow due to the potential gradient acting across phase 1 and the (additional) flux due to the potential gradient acting across phase 2, which gives rise to the viscous drag of the second phase on the first.

The realization that viscous coupling between fluid phases should be incorporated into the immiscible, two-phase flow problem has led a number of researchers (de la Cruz *et al.*, 1983; Whitaker, 1986; Kalaydjian, 1987) to reformulate the immiscible, two-phase flow problem by undertaking a volume averaging of the Navier-Stokes equation.

The equations resulting from this reformulation of the problem show that four independent generalized mobility coefficients are required to define completely the flow characteristics of a particular porous medium-fluid system. Because four, rather than the usual two, mobilities must be determined, an additional experiment must be conducted. Moreover, because viscous coupling effects are known to be significant (Bentsen and Manai, 1993), and because conventional techniques for estimating mobility are based on the assumption that momentum transfer between fluid phases is negligible, it seems clear that neglect of viscous coupling between fluid phases likely introduces a significant amount of model error into effective mobility curves obtained using conventional methods.

The macroscopic pressures in two immiscible phases which are flowing in a porous medium are not, in general, equal in the plane normal to the direction of flow. This is because, as noted by Leverett (1941), the two phases are separated by curved interfaces. As a reasonable approximation, Leverett (1941) assumed that the difference in pressure between the two phases is defined by the static macroscopic capillary pressure, P_c. This assumption enabled Leverett (1941) to postulate that the pressure gradients in two-phase, immiscible flow are related by the capillary pressure gradient, a quantity which depends upon saturation gradient and saturation only, provided the assumption concerning P_c is valid. Implicit in this approach is the assumption that the effect of hydrodynamic forces on the macroscopic pressure difference between two flowing phases is negligible.

However, the difference in macroscopic pressure which exists between two fluids flowing through a porous medium may be sensitive, under certain circumstances, to hydrodynamic effects. This observation is supported by the experimental results of Labastie *et al.* (1980), Bentsen and Manai (1991) and Kalaydjian (1992). In particular, Labastie *et al.* (1980) reported that the difference in macroscopic pressure between two flowing phases was sensitive to rate in fractionally oil-wet porous media, while Bentsen and Manai (1991) reported experimental results from which it can be inferred that hydrodynamic effects are important in the steady-state, countercurrent flow of two immiscible fluids through a homogeneous, water-wet, unconsolidated porous medium. Kalaydjian (1992) has shown experimentally that, in consolidated porous media, the imbibition macroscopic capillary pressure curve is dependent on the ratio of the viscous to the capillary forces.

The realization that hydrodynamic effects could, in some circumstances, be important led Bentsen (1992, 1994a) to construct, starting at the macroscopic scale, a more general pressure difference equation, which included the contribution of these effects. In constructing this equation, it was assumed that the difference in macroscopic pressure could be defined as a linear combination of two types of effects: static effects and dynamic effects.

As a consequence, two experiments, rather than the usual single experiment, are needed to define completely the pressure difference-saturation relationship: one to determine the static capillary-pressure component, and one to determine the hydrodynamic component of the pressure-difference curve.

In addition because, in certain cases, hydrodynamic effects are known to be important (Kalaydjian, 1992; Bentsen, 1994a), and because traditional methods for determining the pressure-difference curve are based on the assumption that hydrodynamic effects are negligible, it appears likely that pressure-difference curves, as conventionally measured, contain significant model error.

As noted above, indirect methods are used, in many cases, to determine the fundamental flow properties needed to describe flow through porous media. Moreover, if one or more of the assumptions underlying the mathematical description of such indirect methods is invalid, significant model error can be introduced into the measured value of the flow property.

The primary purpose of this paper is to explore the impact that neglect of viscous coupling between fluid phases and neglect of hydrodynamic effects have on effective mobility and pressure-difference curves, respectively. A secondary objective is to investigate the effect of flow regime, scale and improper handling of the inlet boundary condition on the effective mobility curves.

These goals are achieved by utilizing experimental data obtained in two earlier experimental studies (Sarma and Bentsen, 1989; Bentsen and Manai, 1991).

1 THEORY

In the analysis that follows, attention is restricted to the stable, colinear, horizontal flow of two immiscible, incompressible fluids through a water-wet, unconsolidated, isotropic and homogeneous porous medium.

Moreover, it is assumed that phase 1 is the wetting phase and that phase 2 is the nonwetting phase.

1.1 Basic equations

Kalaydjian (1987) has shown that, consistent with the assumptions made above, the generalized transport equations for the flow of two continuous phases may be written as:

$$\nu_1 = \lambda_{11}\frac{\partial p_1}{\partial x} - \lambda_{12}\frac{\partial p_2}{\partial x} \tag{1}$$

and

$$\nu_2 = \lambda_{21}\frac{\partial p_1}{\partial x} - \lambda_{22}\frac{\partial p_2}{\partial x} \tag{2}$$

where $\lambda_{ij} = k_{ij}/\mu_j$, i = 1,2. Moreover, the conventional transport equations for two-phase flow, again consistent with the assumptions made above, may be written as:

$$\nu_1 = \lambda_1\frac{\partial p_1}{\partial x} \tag{3}$$

and

$$\nu_2 = \lambda_2\frac{\partial p_2}{\partial x} \tag{4}$$

where $\lambda_i = k_i/\mu_i$, i = 1,2. Based on recently presented experimental results (Bentsen and Manai, 1991, 1993), it may be inferred that, for steady-state, cocurrent flow,

$$\frac{\partial p_1^{\circ}}{\partial x} = R_{12}\frac{\partial p_2^{\circ}}{\partial x} \tag{5}$$

where the quantities measured in a steady-state, cocurrent experiment are designated by a degree symbol (°), and where R_{12} is a weak function of normalized saturation that is introduced to account for the fact that, for horizontal, steady-state, cocurrent flow, the pressure profile for the wetting phase is not parallel to that for the nonwetting phase (Bentsen and Manai, 1991, 1993).

Note that, by setting R_{12} identically equal to one, the pressure profiles become parallel, as is conventionally assumed. Moreover, while R_{12}, in general, may be infinitely nonunique, it appears that R_{12}, as used in this study, is applicable to all types of flow in one-dimensional, homogeneous, water-wet porous media (Bentsen, 1994a). By combining equations (1) to (4), written for a steady-state, cocurrent-flow experiment, it may be shown (Bentsen, 1994b), in view of equation (5), that:

$$R_{12}\lambda_{11} + \lambda_{12} = R_{12}\lambda_1^{\circ} \tag{6}$$

and that

$$\lambda_{22} + R_{12}\lambda_{21} = \lambda_2^{\circ} \tag{7}$$

Equations (1), (2), (6) and (7) comprise a system of four equations in four unknowns, the generalized mobilities λ_{11}, λ_{12}, λ_{21} and λ_{22}. This system of equations may be solved to yield (Liang and Lohrenz, 1994).

$$\lambda_{11} = \frac{\nu f_1 + \lambda_1^{\circ} R_{12}\frac{\partial p_2}{\partial x}}{R_{12}\frac{\partial p_2}{\partial x} - \frac{\partial p_1}{\partial x}} \tag{8}$$

$$\lambda_{12} = \frac{-\nu f_1 R_{12} - \lambda_1^{\circ} R_{12}\frac{\partial p_1}{\partial x}}{R_{12}\frac{\partial p_2}{\partial x} - \frac{\partial p_1}{\partial x}} \tag{9}$$

$$\lambda_{21} = \frac{\nu f_2 + \lambda_2^{\circ}\frac{\partial p_2}{\partial x}}{R_{12}\frac{\partial p_2}{\partial x} - \frac{\partial p_1}{\partial x}} \tag{10}$$

and

$$\lambda_{22} = \frac{-\nu f_2 R_{12} - \lambda_2^{\circ}\frac{\partial p_1}{\partial x}}{R_{12}\frac{\partial p_2}{\partial x} - \frac{\partial p_1}{\partial x}} \tag{11}$$

It is to be noted that equations (8) to (11) degenerate to forms equivalent to those presented by Liang and Lohrenz (1994), provided R_{12} is set identically equal to one.

Moreover, two experiments, one steady-state and one unsteady-state, are required, if these equations are to be used to determine the generalized mobilities.

1.2 Steady-state, countercurrent flow

For steady-state, countercurrent flow, it may be assumed (Bentsen and Manai, 1991, 1993; Bentsen, 1994a) that

$$\frac{\partial p_1^*}{\partial x} = -R_{12}\frac{\partial p_2^*}{\partial x} \tag{12}$$

where the quantities measured in a steady-state, countercurrent flow experiment are designated by an asterisk (*).

By combining equations (1) to (4) written for a steady-state, countercurrent flow experiment, it may be demonstrated (Bentsen, 1994b), in view of equation (12), that

$$\lambda_1^* = \lambda_{11} - \frac{1}{R_{12}}\lambda_{12} \tag{13}$$

and that

$$\lambda_2^* = -R_{12}\lambda_{21} + \lambda_{22} \tag{14}$$

Equations (13) and (14) can be used to determine the effective mobilities that pertain to steady-state, countercurrent flow, provided the generalized mobilities, λ_{ij}, are available, and provided information is available which can be used to determine R_{12}.

1.3 Pure countercurrent imbibition

For pure countercurrent imbibition (Collins, 1961), $\nu_1 + \nu_2 = 0$. This result, together with equations (1) and (2), may be used to show (Bentsen, 1994c) that

$$\frac{\partial p_1'}{\partial x} = -\frac{\lambda_{12}+\lambda_{22}}{\lambda_{11}+\lambda_{22}}\frac{\partial p_2'}{\partial x} \tag{15}$$

where the quantities measured in a pure countercurrent imbibition experiment are designated by a prime (′).

By combining equations (1) to (4), written for a pure countercurrent imbibition experiment, it may be demonstrated (Bentsen, 1994c) that

$$\lambda_1' = \frac{\lambda_{11}\lambda_{22} - \lambda_{12}\lambda_{21}}{\lambda_{12}+\lambda_{22}} \tag{16}$$

and that

$$\lambda_2' = \frac{\lambda_{11}\lambda_{22} - \lambda_{12}\lambda_{21}}{\lambda_{12}+\lambda_{21}} \tag{17}$$

Equations (16) and (17) can be used to determine the effective mobilities that pertain to pure countercurrent imbibition, provided the generalized mobilities, λ_{ij}, are available.

1.4 Steady-state, cocurrent flow

It is convenient now to suppose that

$$\lambda_{11} = C_1\lambda_1^{\circ} \tag{18}$$

and that

$$\lambda_{22} = C_2\lambda_2^{\circ} \tag{19}$$

where, based on the limited amount of data reported in the literature (Bentsen and Manai, 1991, 1993), C_1 and C_2 appear to be constants that depend upon the amount of viscous coupling that takes place across the fluid-fluid interfaces in a porous medium. If it is further assumed (Bentsen and Manai, 1991, 1993) that

$$\frac{\lambda_{11}}{\lambda_{22}} = \frac{\lambda_1^{\circ}}{\lambda_2^{\circ}} \tag{20}$$

then it follows, in view of equations (18) and (19), that

$$C_1 = C_2 = C \tag{21}$$

By combining equation (8) with equation (18), it may be shown, in view of equation (21), that

$$\lambda_1^{\circ} = \frac{-\nu f_1}{C\frac{\partial p_1}{\partial x} + (1-C)R_{12}\frac{\partial p2}{\partial x}} \tag{22}$$

Moreover, by combining equation (11) with equation (19), it may be demonstrated, in view of equation (21), that

$$\lambda_2^{\circ} = \frac{-\nu f_2 R_{12}}{(1-C)\frac{\partial p_1}{\partial x} + CR_{12}\frac{\partial p_2}{\partial x}} \tag{23}$$

It is to be noted that, if the amount of momentum transfer taking place across the fluid-fluid interfaces in a porous medium is negligible, C = 1 and equations (22) and (23) degenerate, as expected, to forms consistent with equations (3) and (4), respectively.

Moreover, if steady-state pressure gradients are employed in equations (22) and (23), it also follows, in view of equation (5), that equations (22) and (23) degenerate to forms consistent with equations (3) and (4) respectively.

That is to say, if steady-state pressure gradients are used in equations (22) and (23), steady-state effective mobilities will be determined, provided values of fractional flow that pertain to steady-state flow are used.

Moreover, if unsteady-state fractional flows and pressure gradients are available, and if data from a steady-state experiment are available so that R_{12} and C can be estimated, then equations (22) and (23) can be used also to determine the effective mobilities that pertain to steady-state, cocurrent flow.

1.5 Pressure-difference equation

Bentsen and Manai have reported (Bentsen and Manai, 1991; Bentsen, 1992) that, for steady-state, countercurrent flow, saturation is invariant along the length of the core, and that, while the magnitude of the pressure gradients is approximately the same, they differ in sign.

That is, the difference in macrosopic pressure, $p_2 - p_1$, varies along the length of the core, which is in contradiction with the assumption that, because saturation is invariant along the length of the core, the difference in pressure, as defined by P_c, should be invariant as well.

Such anomalous pressure behaviour cannot be explained if one assumes that the difference in pressure that exists between two immiscible fluids flowing through a porous medium is defined only by the static capillary pressure.

A possible explanation for such anomalous behaviour may be that the pressure difference between two flowing phases depends not only on the static capillary pressure, but also on hydrodynamic effects.

Because such an explanation is consistent with the experimental results presented earlier (Bentsen and Manai, 1991; Bentsen, 1992), it is supposed that the difference in pressure between two flowing phases, $p_d = p_2 - p_1$, is defined by:

$$p_d = P_c + p_h \tag{24}$$

where P_c is the static macroscopic capillary pressure, and where p_h is the contribution to the difference in pressure between two flowing phases that arises out of hydrodynamics effects. Kalaydjian (1992) has also constructed a pressure difference equation that includes a dynamic term.

However, as discussed later, the physical origin of the dynamic term in equation (24) is thought to be different from that giving rise to the dynamic term in Kalaydjian's pressure-difference equation.

The quantity p_h, at this point, is undefined. However, it can be inferred from the experimental results of Bentsen and Manai (1991) that it may be possible to define the gradient of p_h in terms of an experiment. Taking the partial derivative of equation (24) with respect to x yields:

$$\frac{\partial p_d}{\partial x} = \frac{\partial P_c}{\partial x} + \frac{\partial p_h}{\partial x} \tag{25}$$

Based on the work of Bentsen (1992), it is postulated that

$$\frac{\partial p_h}{\partial x} = (1 - R_{12}) \frac{\partial p_2}{\partial x} \tag{26}$$

Introducing equation (26) into equation (25) leads to

$$\frac{\partial p_d}{\partial x} = \frac{\partial P_c}{\partial x} + (1 - R_{12}) \frac{\partial p_2}{\partial x} \tag{27}$$

where

$$R_{12} = 1 - a(1 - S) \tag{28}$$

and where a is a dimensionless parameter whose magnitude must be determined experimentally (Bentsen, 1992).

It is to be noted that, if the hydrodynamic effects are negligible, a = 0 and R_{12} = 1 and equation (27) degenerates to the form conventionally used.

Equation (27) has been tested experimentally only for one-dimensional, horizontal, steady-state cocurrent and countercurrent flow in homogeneous, unconsolidated, high permeability porous media (Bentsen, 1992).

Consequently, its use in other types of porous media, such as heterogeneous, consolidated porous media, should be undertaken with caution.

1.6 Stability

Buckley-Leverett theory (Buckley and Leverett, 1942) is based on the assumption that the pressure and saturation are uniform at each cross-section along the length of the core (Collins, 1961). If viscous fingers are propagating, such will not be the case.

The stability of a given displacement can be determined by calculating the instability number defined by (Sarma and Bentsen, 1987; Bentsen, 1990):

$$I_{sr} = \frac{M-1-N_g}{N_c}\frac{A_c}{\sigma_{eb}L}C_r(M)\frac{h^2b^2}{h^2+b^2} \qquad (29)$$

where

$$M = \frac{k_{1r}}{\mu_1}\frac{\mu_2}{k_{2i}} \qquad (30)$$

$$N_c = \frac{A_c k_{1r}}{vL\mu_1} \qquad (31)$$

$$\sigma_{eb} = A_c\Phi(1-S_{1i}-S_{2r})w_{cf} \qquad (32)$$

and

$$C_r(M) = \frac{4(M^{5/3}+1)}{(M+1)(M^{1/3}+1)^2} \qquad (33)$$

The displacement is stable provided $I_{sr} \leq \pi^2$ (Sarma and Bentsen, 1987). The function $C_r(M)$ arises out of the fact that a water finger is wider than the contiguous, oppositely directed oil finger (Bentsen, 1985).

2 DATA

To illustrate the use of the equations developed in the section on theory, one must have access to data from a steady-state and an unsteady-state experiment. Ideally, these two sets of data should be acquired using the same sand-fluid system. While such an ideal data set is unavailable, data from two experiments that used the same sand and essentially similar fluids are available.

That is to say, the unsteady-state data used to illustrate the use of the equations are those reported by Sarma and Bentsen (1989a), while the steady-state data needed to augment the unsteady-state data set were taken from the experimental study reported by Bentsen and Manai (1991).

The reader interested in learning more about the equipment, materials and procedures used to acquire these data is referred to the original papers.

In the interests of brevity, only a brief summary of the data essential to this study is reported here. Moreover, only the smoothed curves, which have been fitted to the original data, are reported below.

A summary of the essential experimental data is reported in Table 1. The values of a and C reported in Table 1 were estimated using the steady-state data reported by Bentsen and Manai (1991). The other values reported in Table 1 are based on the unsteady-state data reported by Sarma and Bentsen (1989a).

TABLE 1

Summary of experimental data

Displaced fluid	LAGO
Viscosity of displaced fluid (mPa·s)	4.7
Displacing fluid	water
Viscosity of displacing fluid (mPa·s)	1.0
Length of core (m)	0.985
Thickness of core (m)	0.011
Height of core (m)	0.05
Displacement rate (m^3s^{-1})	6.667×10^{-8}
Absolute permeability (μm^2)	17.17
Porosity (fraction)	0.318
Irreducible saturation to the wetting phase (fraction)	0.07
Residual saturation to the wetting phase (fraction)	0.10
Effective permeability to oil at S_{1i} (μm^2)	10.38
Effective permeability to water at S_{2R} (μm^2)	5.114
Area under the capillary pressure curve (Pa)	1356.6
Width of capillary fringe (m)	0.01
a	0.05
C	0.85

The smoothed pressure gradient curves, corresponding to a time of 1020 seconds, which were used in this study, are depicted in Figure 1. The parameter R_{12}, as a function of S_1, is shown in Figure 2. The steady-state, cocurrent mobility curves employed in this study are displayed in Figure 3. The smoothed fractional flow curve used, again corresponding to a time of 1020 seconds, is depicted in Figure 4.

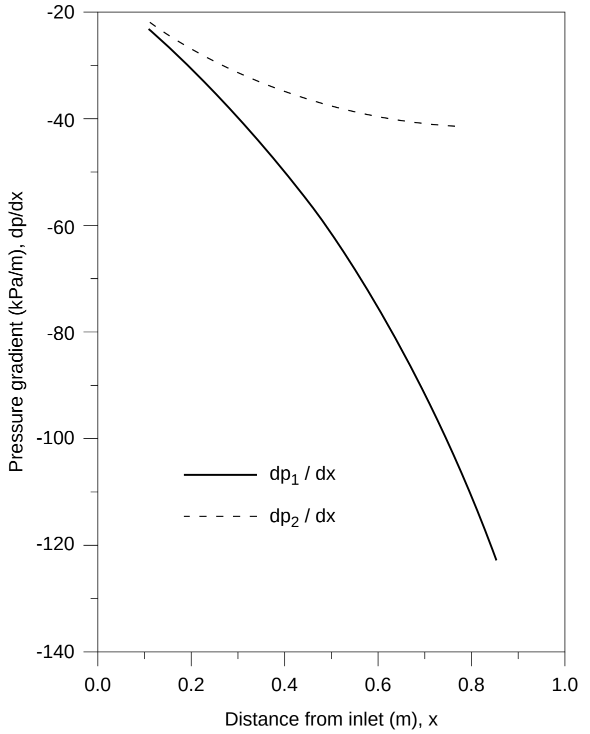

Figure 1

Smoothed pressure-gradient profiles for an unstabilized displacement at 1020 seconds.

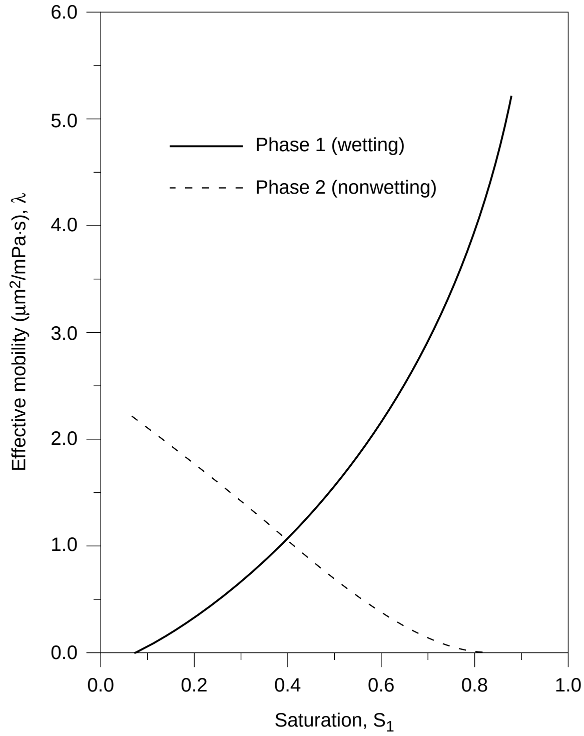

Figure 3

Steady-state, cocurrent mobility curves for phase 1 and phase 2.

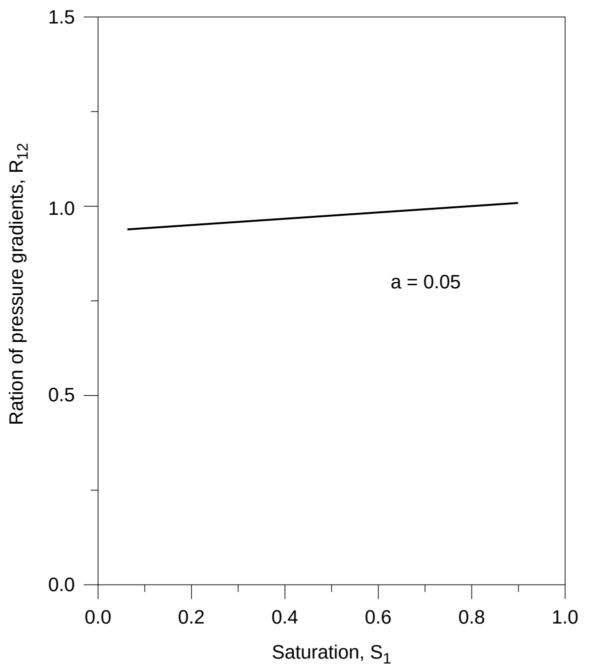

Figure 2

Pressure gradient function for cocurrent flow.

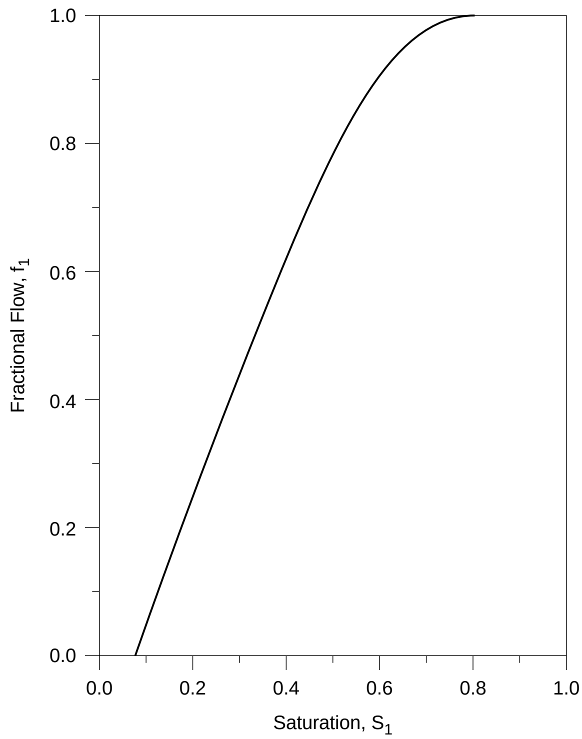

Figure 4

Fractional flow curve for an unstabilized displacement at 1020 seconds.

Finally, the saturation profile (t = 1020 seconds), upon which the analysis is based, is shown in Figure 5.

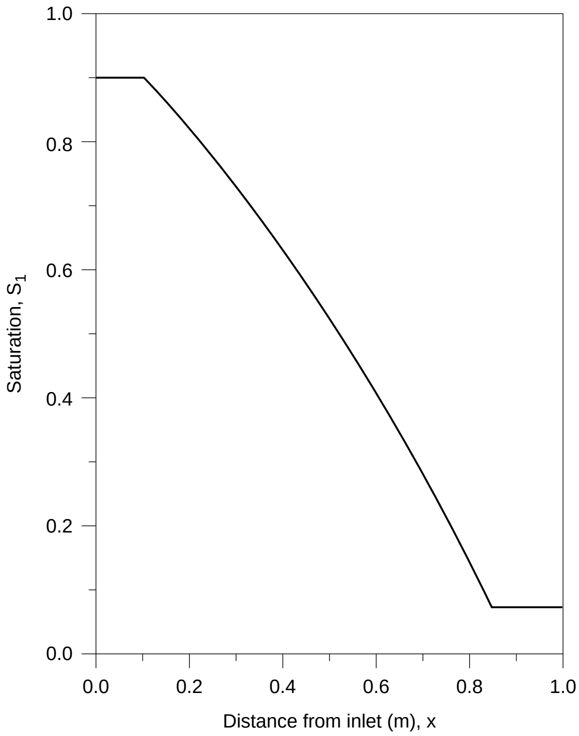

Figure 5

Smoothed saturation profile for an unstabilized displacement at 1020 seconds.

3 RESULTS

The generalized mobilities, λ_i, were estimated using the following procedure. With the help of the parametric equation used to construct Figure 5, the value of the distance x associated with a specific saturation, say S_1^+, was determined.

This value of x, together with the parametric equations used to fit the pressure profiles, was used to estimate the magnitudes of the pressure gradients associated with S_1^+. Then, the magnitude of $f_1(S_1^+)$ was calculated using the parametric equation used to fit the fractional flow curve depicted in Figure 4. Next, the parametric equation used to construct the plot of R_{12} versus S_1 (Fig. 2) was used to determine $R_{12}(S_1^+)$.

Finally the parametric equations, upon which the effective mobility curves presented in Figure 3 are based, were used to determine $\lambda_1^\circ(S_1^+)$ and $\lambda_2^\circ(S_1^+)$, respectively.

This procedure was repeated for each of the values of S_1 used to establish the generalized mobility curves.

Once a complete set of values for each of the parameters needed was in hand, equations (8), (9), (10) and (11) were used to determine λ_{11}, λ_{12}, λ_{21} and λ_{22}, respectively.

The generalized mobility curves generated in this way are shown in Figure 6. It should be noted that the generalized mobility curves depicted in Figure 6 are consistent with the generalized relative permeability curves reported in an earlier experimental study (Bentsen and Manai, 1993).

It is also important to note that, while the curves depicted in Figure 6 are based on data from steady-state and unsteady-state, cocurrent experiments, those reported in the earlier study were based on data from steady-state, cocurrent and countercurrent-flow experiments.

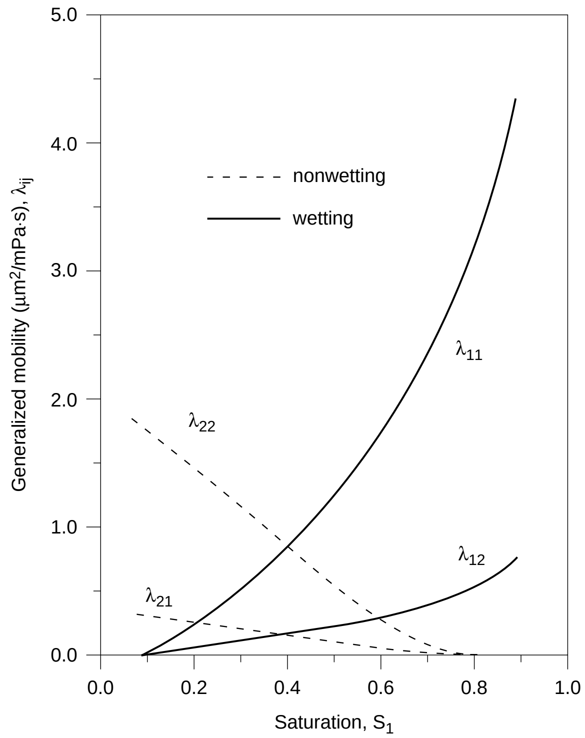

Figure 6

Generalized mobilities for phase 1 and phase 2.

Once the generalized mobility curves are available, it becomes possible to generate the effective mobility curves that pertain to steady-state, countercurrent flow, and to pure countercurrent imbibition. Equations (13) and (14) were used to determine the former curves, while equations (16) and (17) were used to determine the latter curves.

The effective mobility curves for steady-state, cocurrent flow, steady-state, counter-current flow and for pure countercurrent imbibition are compared in Figure 7. Note that the curves shown in Figure 7 are consistent with the relative permeability curves presented in an earlier study (Bentsen, 1994c), which were constructed assuming data were available from steady-state, cocurrent-flow and countercurrent-flow experiments.

Effective mobility depends upon the amount of momentum transfer that takes place across the fluid-fluid interfaces located in a porous medium.

If viscous coupling between fluid phases is negligible, C = 1. Consequently, by setting C = 1, equations (22) and (23) can be used to generate the mobility curves that would pertain, if the viscous drag of one fluid on the other is negligible. Moreover, by setting C = 0.85 (Bentsen and Manai, 1993), equations (22) and (23) can be used also to generate the effective mobility curves that pertain when significant viscous coupling occurs between the two fluids.

The curves for effective mobility that pertain when viscous coupling between phases is negligible (C = 1) are compared, in Figure 8, with those that pertain when viscous coupling effects are significant (C = 0.85). As can be seen from Figure 8, incorporating the effects of viscous coupling between fluids into the analysis results in an increase, at any given saturation, in the wetting-phase mobility, whereas incorporating these effects into the analysis results in a decrease, at any given saturation, in the nonwetting-phase mobility.

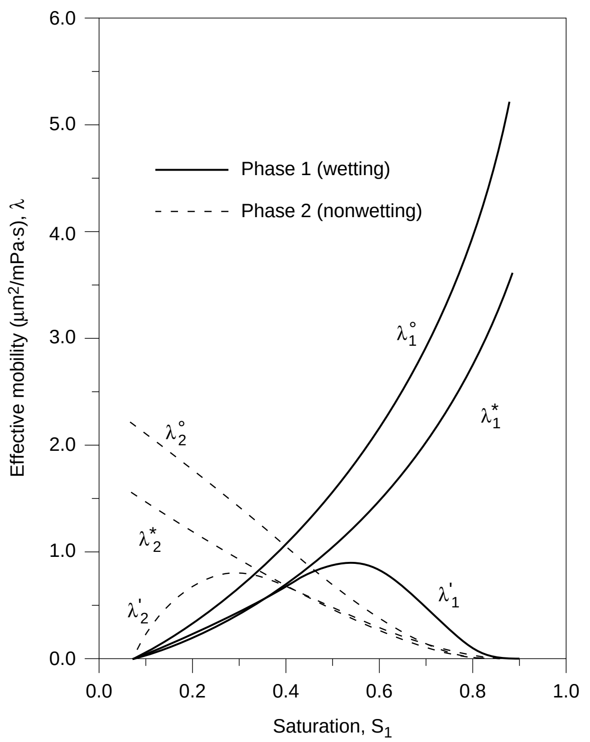

Figure 7

Comparison of effective mobility curves for cocurrent, countercurrent and pure countercurrent flow.

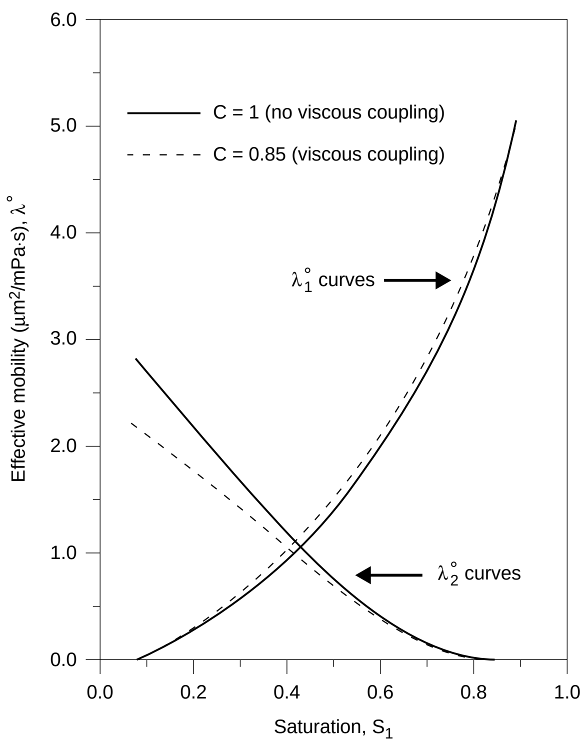

Figure 8

Comparison of effective mobility with and without viscous coupling between fluid phases.

Notice that, while the magnitude of the difference between the two mobility curves, for phase 2, increases as the saturation to phase 2 increases, the magnitude of this difference, for phase 1, increases and then decreases as the saturation to phase 1 increases. The difference between the two phase 2 mobility curves is most pronounced for saturations less than the floodfront saturation.

4 ERROR ANALYSIS

In this section, the impact of neglecting the effect of momentum transfer between fluid phases and hydrodynamic forces is evaluated by determining the relative error incurred by neglecting these effects. To be specific, it is assumed that, when viscous coupling effects are excluded, C = 1, and when they are included, C = 0.85.

Moreover it is assumed that, when the hydrodynamic forces are negligible, a = 0.0 (R_{12} = 1), and when they are not, a = 0.05. The impact of neglecting the viscous drag of one fluid on the other can be evaluated by setting C = 1 in equations (22) and (23). That is, the relative error incurred by setting C = 1 in equation (22) is defined by:

$$\varepsilon_{R1} = \frac{-100(1-C)\dfrac{\partial P_c}{\partial x}}{\dfrac{\partial p_1}{\partial x}} \tag{34}$$

and that in equation (23) by:

$$\varepsilon_{R2} = \frac{100(1-C)\dfrac{\partial P_c}{\partial x}}{R_{12}\dfrac{\partial p_2}{\partial x}} \tag{35}$$

For wetting-phase saturations of 0.07, 0.50 and 0.90, equation (34) predicts that the relative error incurred in equation (22) by neglecting viscous coupling between fluids is 10.1%, 6.28% and 0.983%, respectively. For the same saturations, equation (35) predicts that the relative error incurred in equation (23) by neglecting these effects is –30.8%, –10.8% and –1.05%, respectively.

The relative error in the nonwetting-phase effective mobilities is larger than that in the wetting-phase effective mobilities because the magnitude of the nonwetting-phase pressure gradient is smaller than that of the wetting phase. Moreover, while the relative errors for the wetting phase appear to be significant, the actual errors, as can be seen in Figure 8, are quite small. In this regard, it should be noted that the relative errors for the wetting phase are exaggerated because of the way relative error is defined. This is particularly the case at low values of the wetting-phase saturation, where the magnitude of λ_1° is quite small.

The effect of neglecting the hydrodynamic forces in equation (27) can be ascertained by setting R_{12} = 1. The relative error incurred by setting R_{12} = 1 in equation (27) is defined by:

$$\varepsilon_{Rd1} = \frac{100(1-R_{12})\dfrac{\partial p_2}{\partial x}}{\dfrac{\partial p_2}{\partial x} - \dfrac{\partial p_1}{\partial x}} \tag{36}$$

For a = 0.05, and for wetting-phase saturations of 0.07, 0.50 and 0.90, equation (36) predicts the relative error incurred in equation (27) by neglecting the hydrodynamic effects to be -2.63%, -3.55% and 0.0%, respectively.

These errors are of the same order as the measurement errors for the pressure gradients which were estimated in an earlier study (Sarma and Bentsen, 1990).

For countercurrent flow, the pressure gradients in the wetting and nonwetting phases are opposite in sign. Consequently, in this case, the relative error incurred by neglecting the hydrodnamic effects is defined by:

$$\varepsilon_{Rd2} = \frac{100(1+R_{12})\dfrac{\partial p_2}{\partial x}}{\dfrac{\partial p_C}{\partial x} + (1+R_{12})\dfrac{\partial p_2}{\partial x}} \tag{37}$$

No pressure-gradient data are available for unsteady-state, countercurrent flow. However, the relative error can be determined for steady-state, countercurrent flow.

At steady-state, $\partial P_c/\partial x = 0$; as a consequence, the relative error incurred by neglecting the hydrodynamic effects, as can be seen from equation (37), is 100% for all saturations. That is to say, while neglect of hydrodynamic effects introduces a tolerable amount of error into the analysis for cocurrent flow, such neglect introduces an intolerable amount of error into that for countercurrent flow.

5 DISCUSSION OF RESULTS

5.1 Viscous coupling between fluid phases

In the conventional formulation of the immiscible, two-phase flow problem, it is usual to neglect the transfer of momentum that takes place across the fluid-fluid interfaces located in a porous medium. The model error that arises out of such neglect is incorporated into the conventional mobilities measured in a given type of experiment. As a consequence, as can be seen in Figure 7, different kinds of flow give rise to very different effective mobility curves. That is to say, effective mobility, as conventionally determined, is not a true parameter; rather, it is infinitely nonunique.

It appears that one can still, without the introduction of serious error, use equations (3) and (4) as the basis for reservoir simulation, for example, if two conditions are met. First, if laboratory determined effective mobilities are to be used as data input for the simulator, the mathematical description of both the laboratory method and the simulator must be based on the same set of assumptions. Second, the type of flow in the experiment used to determine λ_1 and λ_2 must be essentially similar to that in the reservoir for which the simulator is to be used. Thus, because most field displacements involve unsteady-state, cocurrent flow, use in a conventional simulator of effective mobilities determined using a traditional unsteady-state, cocurrent-flow method should be acceptable because both the simulator and the unsteady-state method are based implicitly on the assumption that momentum transfer between fluid phases is negligible.

However, use of effective mobilities determined using methods for which the flow regimes in the mobility experiment and the reservoir differ is to be avoided. For example, use in a conventional simulator of effective mobilities obtained using a steady-state, cocurrent-flow method will result in the introduction of error (Fig. 8) into the analysis. This is because mobility curves acquired using a steady-state method include the effects of viscous coupling between fluid phases which is inconsistent with the assumption of negligible viscous coupling upon which conventional reservoir simulators are based.

Similar comments hold with respect to the use of conventionally determine effective mobilities to predict countercurrent flow and pure cuntercurrent imbibition. In particular, it is important to keep in mind that, in fractured reservoirs, waterflooding can lead to both cocurrent and countercurrent flow because of water imbibition into the matrix blocks. That is, when such imbibition takes place, proper account must be taken of the impact of flow conditions on the effective mobilities.

As demonstrated in the section on error analysis, the use of unsteady-state, cocurrent-flow mobilities, which exclude the effect of viscous coupling between fluids, instead of unsteady-state, cocurrent flow mobilities, which include the effect of viscous coupling, can result in relative errors as large as 30% in the magnitude of the effective mobilities. However, it is important to keep in mind that significant error is introduced into the analysis only for saturations less than the floodfront saturation. That is, for saturations greater than the floodfront saturation, the maximal relative error incurred by excluding viscous coupling is about 10%. As a consequence, the introduction of such error is expected to have a significant impact only on the predicted shape of the frontal region of a saturation profile. Moreover, because the actual shape of the frontal region is unimportant in most practical problems, errors arising out of the use of effective mobilities that exclude viscous coupling instead of those that include such coupling are usually tolerable.

5.2 Hydrodynamic effects

Conventional theory for immiscible, two-phase flow neglects the impact that hydrodynamic forces have on the difference in pressure that exists between two fluids flowing through a porous medium. Based on the analysis carried out in the section on error analysis, it appears that such neglect introduces relative errors of the order of a few per cent into the analysis for cocurrent flow, and relative errors of the order of 100 per cent into the analysis for countercurrent flow. Because the errors for cocurrent flow are of the same order of magnitude as the measurement errors for the pressure gradients, it appears that, from a practical point of view, neglect of hydrodynamic effects is acceptable for cocurrent flow. However, because of the very large relative errors incurred, such neglect is not acceptable for countercurrent flow.

Neglect of the hydrodynamic forces is also a concern from a theoretical point of view. To understand why this should be the case, it is necessary to consider the colinear flow of two immiscible fluids through a porous medium. For horizontal, cocurrent flow, equation (27) may be written, in view of the definition for p_d and equation (28) as:

$$\frac{\partial p_2}{\partial x} - \frac{\partial p_1}{\partial x} = \frac{dP_c}{dS_1}\frac{\partial S_1}{\partial x} + a(1-S)\frac{\partial P_2}{\partial x} \qquad (38)$$

At steady-state, S_1 is invariant with x. As a consequence, equation (38) becomes, upon rearrangement,

$$\frac{dp_1^{\circ}}{dx} = [1 - a(1-S)]\frac{dp_2^{\circ}}{dx} \qquad (39)$$

Equation (39) is consistent with equation (5).

The same approach may be followed to obtain the analogous equation for countercurrent flow. However, when doing so, it is necessary to keep in mind that, for countercurrent flow, the function R_{12} is the negative of that which pertains for cocurrent flow (Bentsen, 1992). This is the case because, when fluids flow in opposite directions, the pressure gradients are opposite in sign. Thus, following the approach taken above, and keeping in mind the sign change for R_{12}, it may be shown that:

$$\frac{dp_1^{*}}{dx} = -[1 - a(1-S)]\frac{dp_2^{*}}{dx} \qquad (40)$$

Equation (40) is consistent with equation (12).

Thus, if the hydrodynamic forces are neglected (a = 0), the pressure gradients for steady-state, cocurrent flow are equal (eq. (39)), while for steady-state, countercurrent flow the magnitude of the pressure gradients is the same, but they differ in sign (eq. (40)). If the hydrodynamic forces are included in the analysis (a > 0), the effect is to cause the magnitude of the pressure gradients to differ slightly for both cocurrent and countercurrent flow. These observations are consistent with what has been determined experimentally (Bentsen and Manai, 1991).

It is to be noted that assuming that the difference in pressure between two flowing phases is defined by the static capillary pressure (equivalent to setting a = 0 in equation (38)) introduces very little error into the analysis for cocurrent flow. However, such an approach is to be avoided in the case of steady-state, countercurrent flow. This is because such an assumption not only introduces very large errors into the analysis for countercurrent flow, but also makes it impossible to explain why the pressure gradients should be opposite in sign. That is, given that, for steady-state, countercurrent flow, saturation is invariant with distance (Bentsen and Manai, 1991), the pressure difference between two flowing phases cannot be defined by the static capillary pressure when the two fluids are flowing in opposite directions.

The results presented in this paper are based on experimental work undertaken in homogoneous, high permeability, unconsolidated porous media. The experimental results presented by Kalaydjian (1992) suggest that the impact of hydrodynamic forces might be much more important in heterogeneous, consolidated porous media than is the case for homogeneous, unconsolidate porous media. In unconsolidated porous media, it has been found that, within experimental error, the macroscopic capillary pressure measured under dynamic conditions seems to be the same as that measured under static conditions (Sarma and Bentsen, 1989a). Moreover, again within experimental error, the macroscopic capillary pressure appears to be independent of flow regime. That is to say, for a given sand-fluid system, the same macroscopic capillary-pressure curve appears to apply to unstabilized flow (capillary forces greater than viscous forces; Sarma and Bentsen, 1989a), stabilized stable flow (capillary and viscous forces approximately in balance; Sarma and Bentsen, 1990) and unstable flow (viscous forces greater than capillary forces; Sarma and Bentsen, 1989b).

Finally, within experimental error, the same macroscopic capillary-pressure curve, albeit defined in a somewhat different way, seems to apply to both cocurrent and countercurrent, steady-state flow (Bentsen and Manai, 1991). In contrast, Kalaydjian (1992) has shown that, in consolidated porous media, the imbibition, macroscopic capillary-pressure curve is dependent on the ratio of the viscous to the capillary forces.

Moreover, the effect of the ratio of the viscous to the capillary forces was found to be more marked in the limestone sample, which had a double porosity, than in the sandstone sample, which had a less complex pore structure. Thus, the impact of model error on the measurement of flow properties appears to be a more serious problem in heterogeneous, consolidated porous media than is the case for homogeneous, unconsolidated porous media. The pressure-difference equation constructed by Kalaydjian (1992) is composed of two parts; a static term and a dynamic term. Moreover, the dynamic term has been shown to be proportional to the time derivative of the saturation. A similar theoretical result has been found by de la Cruz, *et al.* (1995).

Consequently, under steady-state conditions the difference in macroscopic pressure between two fluids flowing through a porous medium should be defined by the static, macroscopic capillary-pressure, a result that is in contradiction to the experimental results presented by Bentsen and Manai (1991) for steady-state, countercurrent flow. This suggests that the hydrodynamic effects observed by Bentsen and Manai (1991) and those observed by Kalaydjian (1992) have their origins in different types of physical phenomena.

It appears that the hydrodynamic effects observed by Bentsen and Manai (1991) are due solely to capillary forces (Bentsen, 1992, 1994a), while those reported by Kalaydjian (1992) arise out of the interaction between capillary and viscous forces. Further experimentation is needed to explain completely these differences.

5.3 Stability

As noted by Bentsen (1985), the flow regime in a laboratory experiment usually differs significantly from that which pertains in the field. To illustrate this point, let us suppose that the mobility curves reported in this study were measured so that they could be used as a part of a data set for a waterflood study of a particular field. The data reported in Table 1, together with equations (29) to (33), may be used to show that, under laboratory conditions, the instability number, I_{sr}, equals 1.13. That is to say, the displacement upon which the data reported in this study are based was a stable displacement. Let us suppose now that, under field conditions, all of the parameters remain the same, except for the velocity and the size of the system.

If under field conditions, $v = 3.528 \times 10^{-6}$ m/s (0.3 m/d), L = 280 m, h = 6 m and b = 140 m, I_{sr} may be determined to be 1.03×10^4. That is, despite the fact that the field velocity is approximately 35 times smaller than the laboratory velocity, the field displacement is pseudostable (Sarma and Bentsen, 1987). Thus because the effective mobility curves that pertain to stable flow differ significantly from those that pertain to unstable flow (Sarma and Bentsen, 1989b), it would be inapproriate to use the laboratory determined mobility curves as a part of the data base for the waterflood study.

5.4 Inlet boundary condition

If a displacement is unstabilized, or if it is unstable, the saturation at the inlet end of the core may not increase to its maximal value by the time breakthrough of the injected fluid is achieved (Sarma and Bentsen, 1989a). Recently, Shen *et al.* (1994) have presented results that show that the normalized inlet saturation is dependent on the dimensionless time, τ/N_c, and that it rises to unity in finite time. This study shows that, for large mobility ratios, and/or large capillary numbers, a substantial number of displaceable pore volumes have to be injected before the normalized inlet saturation reaches unity. This result may be illustrated as follows. The data reported in Table 1, together with equation (31), may be used to show that, under the laboratory conditions reported in this study, $N_c = 0.058$. If one assumes that the dimensionless time, τ/N_c, required for the inlet saturation to rise to unity is equal to 6.8, it follows that 0.3944 recoverable pore volumes of fluid must be injected before the inlet saturation achieves its maximal value.

Using the data reported in Table 1, the time needed for the inlet saturation to rise to its maxmal value may be estimated to be about 850 seconds, a value that is consistent with the saturation profiles and times reported in Figure 3 in an earlier experimental study (Sarma and Bentsen, 1989a). Suppose now that the length of the core had been 0.05 rather 0.985 m, a length more representative of the cores used in relative permeability studies. If such is the case, $N_c = 1.15$ and 7.82 recoverable pore volumes have to be injected before the inlet saturation rises to its maximal value. Thus, if one assumes, as is usually done, that the inlet saturation instantly rises to its maximal value, serious error will be introduced into the analysis normally used to determine conventional effective mobility curves. This is because the pressure drop history, upon which the unsteady-state methods for estimating effective mobilities is based, is related directly to how the inlet saturation varies (Shen and Ruth, 1996).

5.5 Impact of scale

Effective mobilities are usually measured so that they can form a part of the data base for a reservoir-scale simulation. Because the mobilities are normally obtained by flooding homogeneous, macroscopic cores, a question arises as to whether such mobilities are the same as those that pertain to the heterogeneous, megascopic grid block from which the macroscopic core was obtained. To answer this question, Haldorsen (1983) undertook a number of numerical experiments.

As input, Haldorsen (1983) employed the set of linear relative permeability curves displayed in Figure 9.

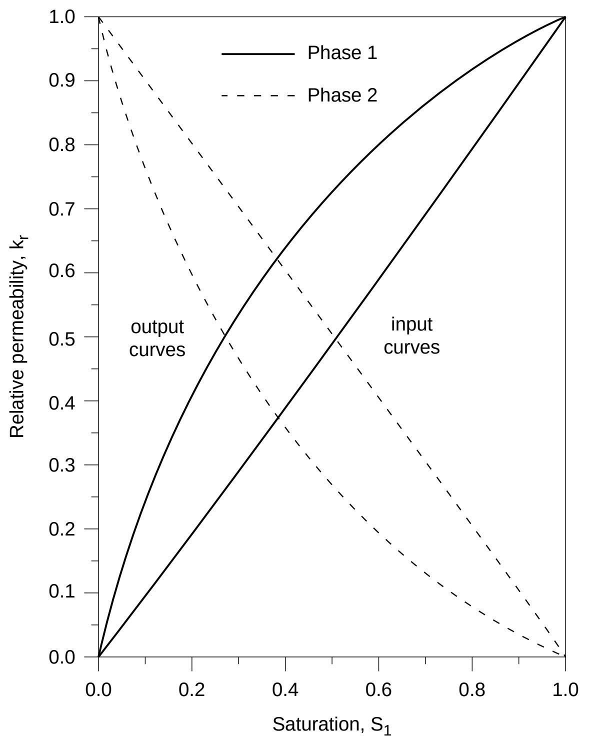

Figure 9

Comparison of linear (input) and nonlinear (output) relative permeability curves. After Haldorsen (1983).

It was assumed that these curves were obtained by displacing phase 2 by phase 1 in a homogeneous sandstone core. As a matter of convenience, the residual saturations were set to zero. The viscosity of both phases was set equal, and was assumed to be independent of pressure. Moreover, the densities of the two phases were taken to be equal. Finally, for all of the numerical experiments, the flooding was in the vertical direction.

The method of Johnson *et al.* (1959), as modified by Jones and Roszelle (1978), together with the pressure and production history predicted by the simulator, was used to generate the nonlinear (output) relative permeability curves depicted in Figure 9. As can be seen from Figure 9, the megascopic (output) relative permeability curves differ significantly from the macroscopic (input) curves. This result is due to the effects of numerical dispersion.

The ouput curves can be made identical to the input curves by reducing the grid-block size sufficiently. However, to do so is not practicable in a field-scale simulation. As a consequence, it appears that it is necessary, in most practical problems, to adjust the shape of the input relative permeability curves in order to compensate for the effects of numerical dispersion. Note that the magnitude of the adjustment depends on the size of the grid blocks used in the field-scale simulation.

6 CONCLUSIONS

Commonly, indirect methods are used to determine the fundamental flow properties needed to describe flow through porous media. In this study, it is shown that, if the assumptions upon which the indirect method is based are invalid and/or if they are not well met, significant model error can be introduced into the measured value of the flow property. In particular, it is shown that effective mobility curves that include the effect of momentum transfer across fluid-fluid interfaces differ significantly from those that do not include such transfer. Moreover, it is demonstrated that the conventional effective mobilities that pertain to steady-state, cocurrent flow, steady-state countercurrent flow and pure countercurrent imbibition differ significantly. That is to say, traditional effective mobilities, as conventionally measured, are not true parameters; rather they are infinitely nonunique.

In addition, it is shown that, while neglect of hydrodynamic forces introduces a tolerable level of error into the pressure-difference curve for cocurrent flow in unconsolidated porous media, such neglect introduces an intolerable level of error into the pressure difference curve for countercurrent flow in such porous media. Moreover, such neglect makes it impossible to explain why the pressure gradients differ in sign for steady-state countercurrent flow.

This study also points out that the flow in a laboratory core, because ot its small size, is usually stable, while flow in the field, because of its large size, is usually unstable. This is important because the effective mobility curves that pertain to stable flow differ significantly from those that pertain to unstable flow. Moreover, as shown in this study, the inlet boundary condition used in an unsteady-state method for determining effective mobilities is also important. In particular, if the core used in the mobility experiment is short, it may take many pore volumes of injection before the inlet saturation rises to its maximal value, which is in contradiction with the usual assumption that the inlet saturation immediately rises to its maximal value. Consequently, improper handling of the inlet boundary condition can introduce a significant amount of model error into the calculated values of the effective mobilities.

Finally, it is pointed out that, because effective mobilities are measured on macroscopic cores for use in megascopic grid blocks, it may be necessary to adjust the shape of the mobility curves to compensate for the effects of numerical dispersion.

NOMENCLATURE

Roman letters

a	parameter in equation (29)
A_c	area under the capillary pressure curve
b	width of the porous medium
C	parameter defined by equation (22)
C_1	parameter in equation (19)
C_2	paramater in equation (20)
$C_r(M)$	parameter defined by equation (34)
f_i	fractional flow to phase i; i = 1.2
h	thickness of porous medium
I_{sr}	dimensionless velocity (instability number) for a rectangular system
k_i	effective permeability to phase i; i = 1.2
k_{ij}	generalized effective permeability to phase i; i, j = 1.2
k_{1r}	effective permeability to phase 1 at the residual saturation to phase 2
k_{2i}	effective permeability to phase 2 at the irreducible saturation to phase 1
L	length of the porous medium
M	end-point mobility ratio
N_c	capillary number
N_g	gravitational number
p_d	$p_2 - p_1$ = difference in macroscopic pressue between two flowing phases
p_h	hydrodynamic contribution to the difference in macroscopic pressure which exists during flow
p_i	pressure for phase i; i = 1.2
P_c	macroscopic static capillary pressure
q	volumetric flow rate
R_{12}	function relating the pressure gradient in phase 1 to that in phase 2
S	$(S_1 - S_{li})\ (1 - S_{li} - S_{2r})$ = normalized saturation
S_i	saturation of phase i; i = 1.2
S_{li}	irreducible saturation to phase 1
S_{2r}	residual saturation to phase 2
t	time
ν	total velocity
ν_i	Darcy velocity to phase i; i = 1.2
w_{cf}	width of capillary fringe
x	distance in direction of flow.

Greek letters

ε_R	relative error
λ_i	k_i/μ_i = effective mobility of phase i; i = 1.2
λ_{ij}	k_{ij}/μ_j = generalized mobility of phase i; i, j = 1.2
μ_i	viscosity of phase i, i = 1.2
σ_{eb}	bulk frontal tension
τ	$qt/(A\phi L(1 - S_{1i} - S_{2r}))$ = dimensionless time
ϕ	porosity.

Subscripts

c	capillary
d	difference
g	gravity
h	hydrodynamic
i	irreducible.

Superscripts

°	steady-state, cocurrent flow
*	steady-state, countercurrent flow
′	pure countercurrent imbibition
+	particular value.

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