

Transmissibility Corrections and Grid Control for Shale Gas Numerical Production Forecasts

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Résumé — Corrections de transmissivités et contrôle des maillages pour les simulations numériques de production en faible perméabilité — En contexte de roche mère, la perméabilité effective du milieu est si basse que la nature de l'écoulement demeure principalement transitoire, même après des mois ou des années de production. A l'aide d'un simulateur numérique (maillage non-structuré, volumes finis) nous montrons que la non-linéarité du champ de pression au voisinage des puits horizontaux multi fracturés doit être prise en compte lors des prévisions de production.

Un premier exemple, basé sur un PVT (*Pressure Volume Temperature*) linéaire, montre que les transmissivités classiques (linéaires) conduisent à surestimer la production de l'ordre de 5 % par rapport à la réponse analytique. Nous proposons une nouvelle approche de calcul des transmissivités, basée sur des intégrations numériques successives des distributions de points sources. Les résultats prennent en compte la non-linéarité du champ de pression et les interférences entre fractures. La précision des prévisions de production est alors nettement améliorée.

Avec un PVT de gaz réel, les effets non-linéaires deviennent encore plus critiques à proximité des puits. Alors que les solutions analytiques sont insuffisantes dans ce cas, les simulations numériques sont plus précises, à condition toutefois que les maillages utilisés soient suffisamment fins. Afin de réduire les temps de calcul, les simulations de production à long terme sont en général effectuées sur des grilles grossières dont les transmissivités sont corrigées par des méthodes éprouvées de mise à l'échelle. Nous montrons néanmoins que la taille du maillage doit être soigneusement adaptée lorsque la perméabilité est faible. Une méthode d'ajustement automatique de la grille est testée qui prend en compte la perméabilité et la résolution temporelle du problème.

Abstract — Transmissibility Corrections and Grid Control for Shale Gas Numerical Production Forecasts — *In the context of shale gas production, the very low effective permeability of the formation leads to flowing conditions that are essentially transient. Even after months or years of production, the pressure drop remains mainly localized around the hydraulic fractures. Using an unstructured grid, finite-volume simulator, we show that the non-linear nature of the pressure field around horizontal wells with multiple hydraulic fractures can have a non-negligible impact on shale gas production forecasts.*

We first show a very simple synthetic production example with a purely linear PVT (Pressure Volume Temperature). In this case, standard (linear) transmissibility derivations overestimate the forecast after 10 years by 5%, compared to the analytical solution. We propose a new approach for transmissibility derivations, based on numerical integrations of source point solutions. Resulting transmissibility values account for the strong non-linearity of the pressure field in the vicinity of the fractures and for fracture interferences. As a consequence, forecasts are significantly improved.

With a real gas PVT, non-linear effects become even more critical in the vicinity of the well. While analytical solutions only partially account for these effects, numerical simulations are more accurate, provided that the grid is fine enough. In order to reduce the computational cost, long-term simulations are usually performed on a coarser grid, with coarse transmissibility corrections obtained from near-well upscaling techniques. We show that even if near-well numerical upscaling is extremely robust for conventional problems, the choice of an optimal simulation grid size becomes essential for shale gas. A recently proposed automatic adjustment of the grid to the considered problem (including permeability and time resolution) is tested.

INTRODUCTION

Data history matching and production forecasting in shale gas context raises new challenges linked to the low-permeability, fully transient context. Recently, a complete workflow has been proposed and tested [1, 2]. As described in the cited papers, the workflow starts with the simplest methods, such as diagnostic plots and straight line analysis, and progressively includes more sophisticated models in order to account for increasing physical complexity. The last two main steps and most advanced options of the workflow are:

- a transient, analytical model for horizontal wells with multiple hydraulic fractures. This model accounts for interferences between hydraulic fractures and allows for various fracture flow models (infinite conductivity, finite conductivity or uniform flux). However, it does not account for the non-linearity of gas PVT properties. Also, it cannot account for advance effects such as desorption, multiphase flow or unconsolidation, etc.;
- a numerical model including a large range of physical complexity, such as non-linear PVT, complex fractures geometry, heterogeneity, non-consolidation and desorption. Although this model is the one that should ultimately be used for forecasting, it potentially involves so many unknowns that it should be used at the end of the analysis process only, *i.e.* once the main unknowns have been significantly constrained by diagnostic tools and analytical interpretation.

At each step of the workflow, the results of the last analysis are used as a starting point for the next model, which includes additional physical effects. At each stage, results and forecasts are checked, and the validity/influence of any new sophistication can be analyzed in order to ensure proper update of the different parameters.

It is obvious that at each stage, the results of the different methods must be coherent in order to ensure the overall robustness of the complete analysis. In particular, under the same assumptions, different methods should give similar results.

The objective of this paper is to focus on the comparison between the analytical, multiple fractures horizontal well model and the numerical model, trying to explain and correct existing differences. We show that the transient, non-linear

nature of the pressure field around fractures can challenge the classical assumptions of the numerical model. As a consequence, both the transmissibility derivations and the automatic gridding procedure were adapted.

This validation step under simplistic assumptions (simple PVT and geometry, no desorption, etc.) is essential to the integration of the numerical model in the workflow, *i.e.* just before adding further complexity. This should prevent tuning fudge parameters in the full-physics model to compensate for uncontrolled, purely numerical effects, etc.

1 PRESSURE TRANSIENT AND NON-LINEAR EFFECTS

Let us consider a simple reservoir, as described in Table 1 and Figures 1 and 2. For the numerical model, we use a finite-difference simulator with a Voronoi grid, allowing

TABLE 1
Reservoir and well properties

Reservoir data	
Initial reservoir pressure (psia)	5 000
Reservoir temperature (°F)	112
Reservoir radius (ft)	10 000
Net pay (ft)	100
Porosity (%)	10
Swi (%)	0
Gas specific gravity	0.65
Well data	
Horizontal well length (ft)	4 000
Horizontal well position	centered
Horizontal well skin factor	0
Number of fractures	20
Fractures half-length X_f (ft)	250
Fractures position	centered
Fractures penetration	full
Fractures conductivity F_c (md.ft)	1 000
Well flowing pressure (psia)	250
Completion type	cased hole

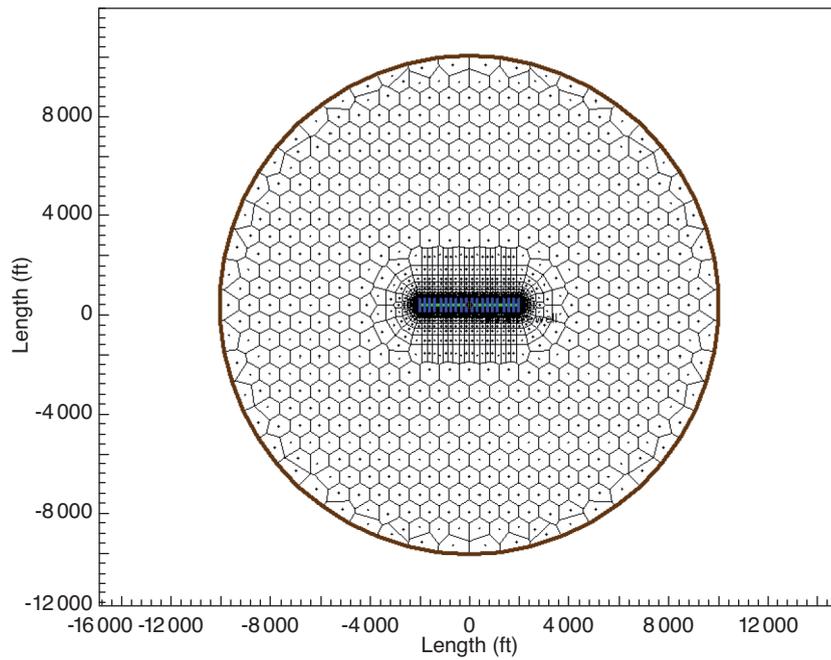


Figure 1
 Voronoi grid for the reservoir described in Table 1.

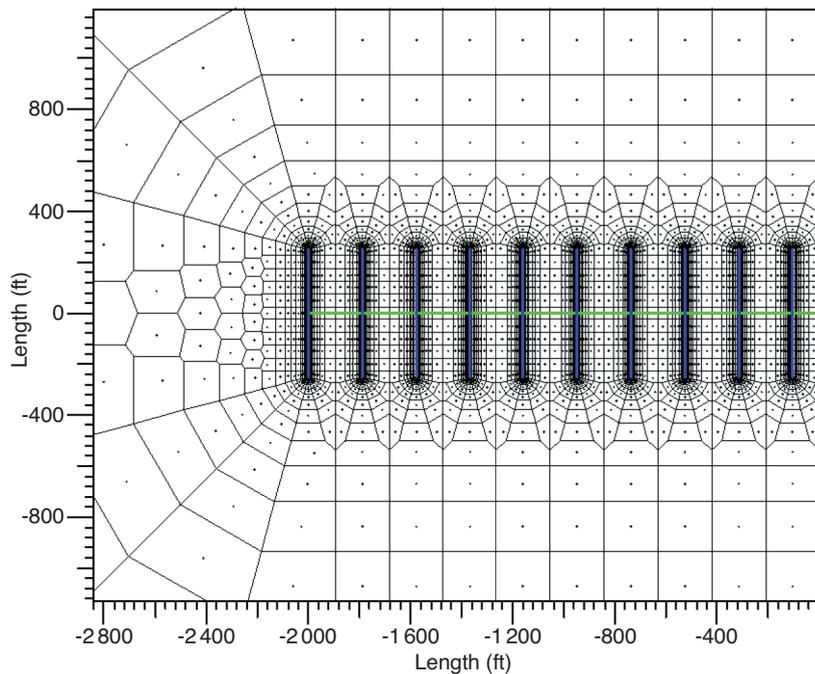


Figure 2
 Voronoi grid (close-up on hydraulic fractures).

smooth grid refinement close to the fractures. By default, the size of the finest cells is automatically set to a couple of times the width of the fracture and a default cell size progression

ratio of 1.4 is applied as one moves away from the fracture, until the constant size “back grid” cells (hexagons) are reached.

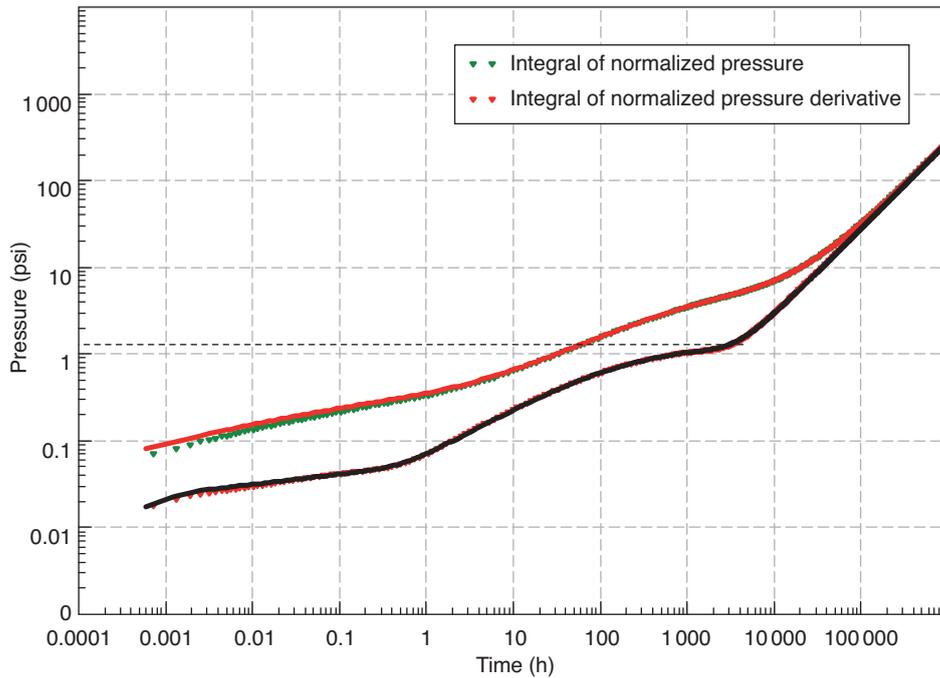


Figure 3

Comparison of pressure and pressure derivative curves between analytical (markers) and numerical (continuous lines) results. $k = 30$ mD.

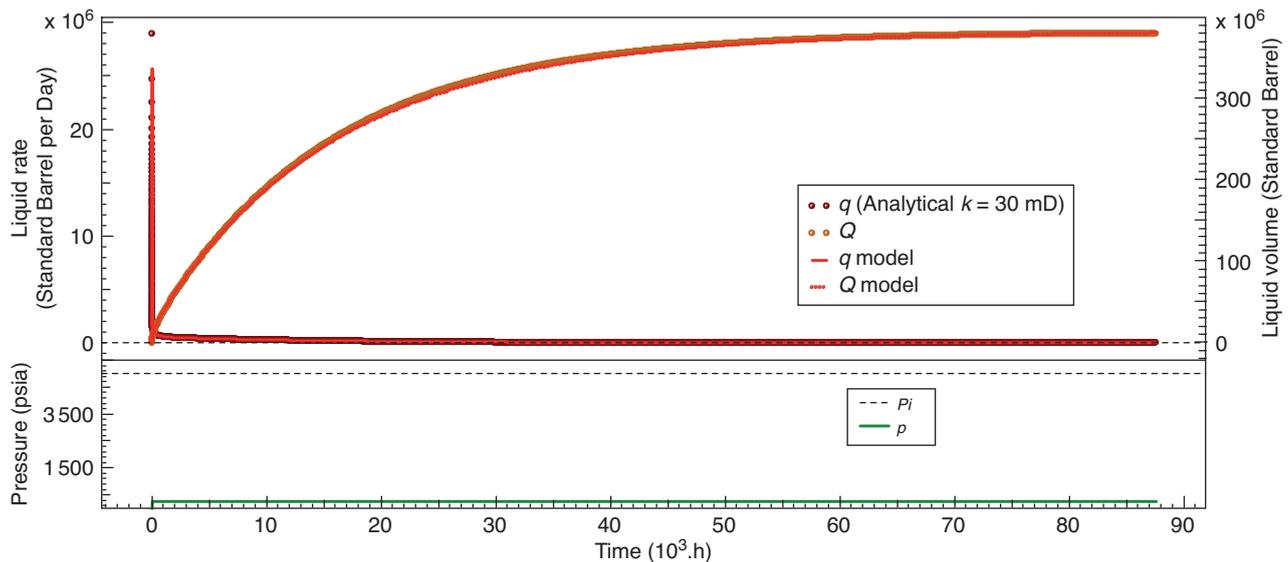


Figure 4

Comparison of analytical (markers) and numerical ("model") forecasts. $k = 30$ mD.

As a starting point, let us further assume that the gas PVT has constant viscosity and compressibility, and that the permeability is homogeneous, with a conventional value $k = 30$ mD.

The reservoir is produced at constant well pressure $P = 250$ psia for ten years. As can be seen from Figures 3

and 4, the match between the analytical model and the numerical model is excellent, both for early-time transient analysis and long-term forecasting.

Let us now reduce the permeability of the medium, so that $k = 10^{-4}$ mD.

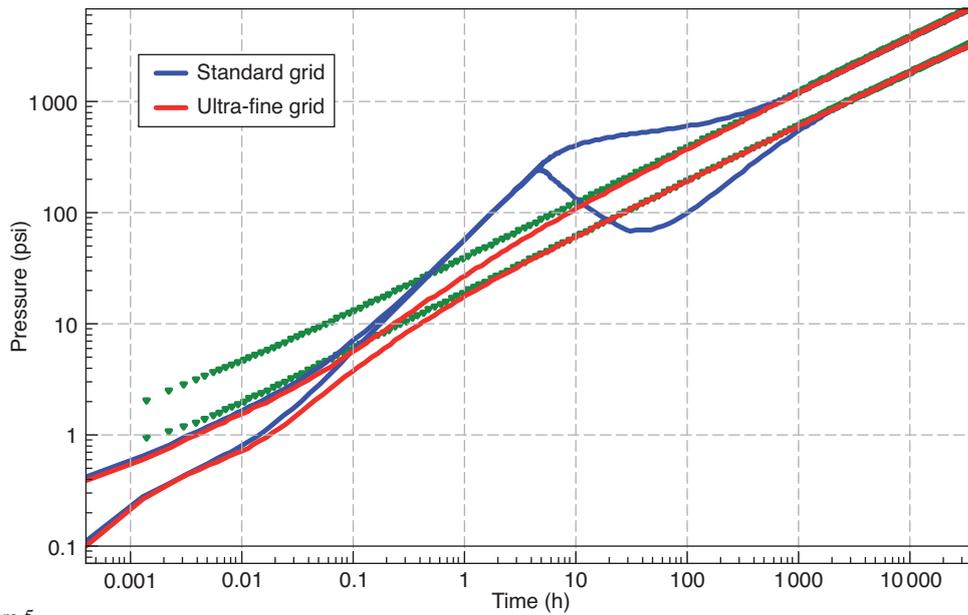


Figure 5

Comparison of pressure and pressure derivative curves between analytical (markers) and numerical (continuous lines) results. $k = 10^{-4}$ mD.

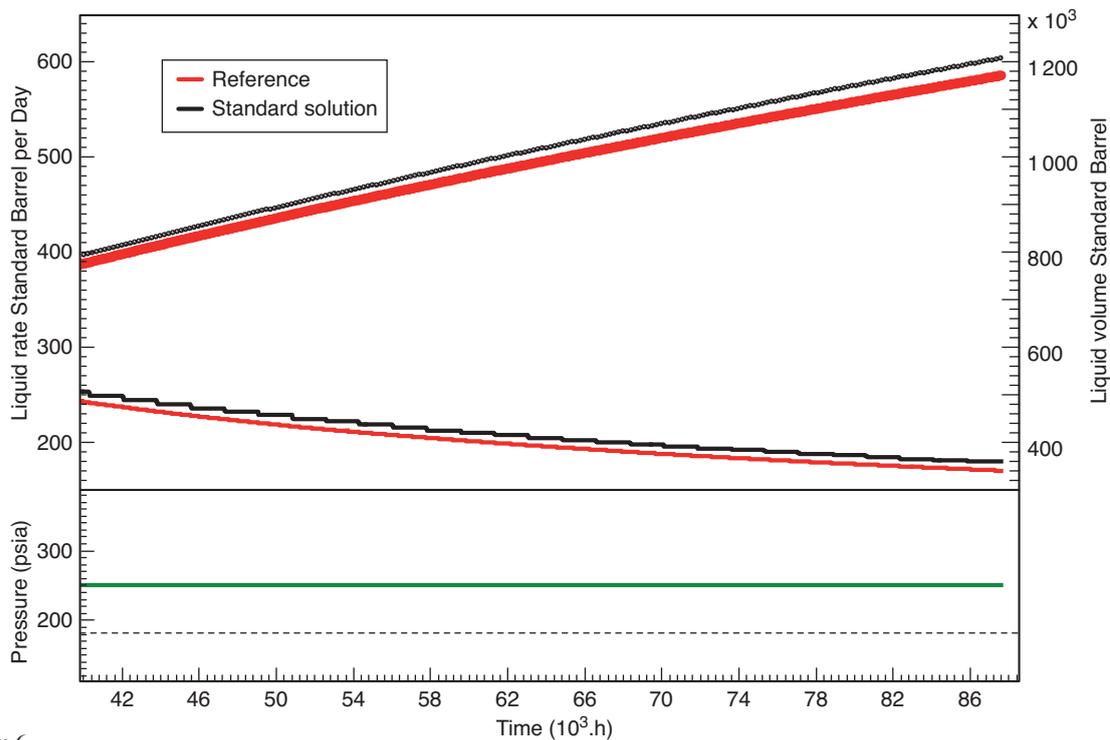


Figure 6

Comparison of analytical (markers) and numerical ("model") forecasts. $k = 10^{-4}$ mD.

It is worth recalling here that reducing k by several order of magnitudes is quite similar to zooming on short times with a conventional case. Small k will hence amplify the importance of transient effects, in a zone where most

numerical errors typically appear. Indeed, in Figures 5 and 6, we now see that several discrepancies arise, both at early time (loglog plot Fig. 5) and at late time (production forecast, Fig. 6).

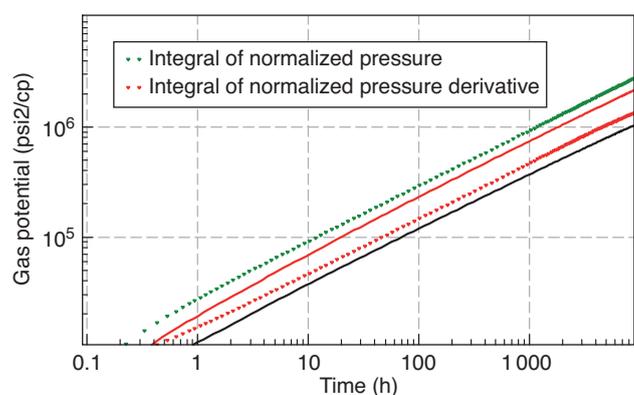


Figure 7

Comparison of pressure and pressure derivative curves between analytical (markers) and numerical (continuous lines) results for a non-linear gas PVT. $k = 10^{-4}$ mD.

First, as visible from Figure 5, the early-time behavior is completely lost. Part of this discrepancy is explained by the compressibility of the fluid inside the fracture, which is accounted for by the numerical model only. However, as was already noticed in [1], the derivative curve obtained with the standard grid shows a purely numerical, “double-porosity looking” effect. This has to be related to the size of the first rings of cells around the fractures. Although the standard size was quite sufficient to capture the early-time behavior for a conventional range of permeability values, we see that it cannot be set irrespectively of the permeability field in nano to micro Darcy permeability formations. Indeed, reducing the grid size to millimeter scale in the vicinity of the fracture solves the problem (ultra-fine grid in Fig. 5). A practical objective of this study is to find the best resolution depending on the context, without having to systematically use the finest grid. This will be developed in Section 5, dedicated to grid refinement control.

Second, the long-term forecasts can be significantly deviating (Fig. 6). After 10 years, the cumulative predicted by the numerical model can be up to 5% greater than the analytical prediction in some cases. This is related to the strong non-linear behavior of the pressure field in the vicinity of the fractures. As a consequence of this effect, the standard linear assumption behind transmissibility derivations cannot hold anymore, although the grid is very fine and k -orthogonal. This phenomenon was negligible with a conventional permeability. With shale gas, however, the pressure drop remains localized in the vicinity of the fractures even after years of production, and non-linear effects cannot be neglected anymore.

Finally, let us consider the non-linear nature of gas PVT properties. As previously noticed in [1], the high compressibility of the gas in the vicinity of the fractures increases the

overall productivity. This effect can be accounted for by the numerical model only. Indeed, by assuming a constant $\mu_g \cdot c_g$ product (gas viscosity \times compressibility), analytical calculations systematically underestimate the productivity in this context (Fig. 7), leading to significantly lower cumulatives. However, in order to fully rely on the numerical answer, we must ensure that the numerical model correctly captures the variations of c_g and μ_g close to the fractures, *i.e.* that the grid is fine enough.

As a consequence of these observations, we decided to develop a new transmissibility derivation algorithm, which should account for non-linear pressure effects and ultimately reduce the discrepancy between analytical and numerical results under similar assumptions. Furthermore, an automatic adjustment of the grid (through numerical upscaling) to the formation permeability and desired time-scale resolution was implemented, following the observations made in [3]. Its purpose is to solve the early-time deviation due to the resolution of the model and better constrain the handling of gas compressibility in the vicinity of the fractures, when non-linear PVT is considered.

2 TRANSMISSIBILITY CALCULATIONS

The problem of correctly modeling fluid flow in the vicinity of the well is a difficult task. This is due to the fact that the transmissibilities are computed based on the gridding geometry, assuming a predefined simple pressure behavior around each cell face. These derivations are usually made assuming linear, geometric or logarithmic pressure evolution away from the well and account for pressure variation on cell faces through-out simple geometrical correction. However, they do not account for pressure variation along the faces themselves. This usually leads to satisfactory results for standard well geometry. For complex 3D geometries, however, neglecting the complex variation of the pressure profile along the faces may lead to incorrect flow representation. In order to correctly model the flow in this region, local pressure variations have to be taken into account. This effect becomes predominant for low permeability problems, where pressure variations are locally very important.

Several authors have investigated this problem by means of simple analytical solutions. This is possible as long as the pressure field is 2D and is limited by simple well geometry. For complex 3D problems, if the fluid is assumed incompressible, a potential solution has to be found numerically. Lee [4, 5] introduced a boundary integral representation for the pressure field around the wellbore and later used the slender body theory in order to correct the production index. A similar method was developed by Ding [6, 7] and extended to the near wellbore region to correct well index and the grid transmissibilities in the near wellbore region. Their integral

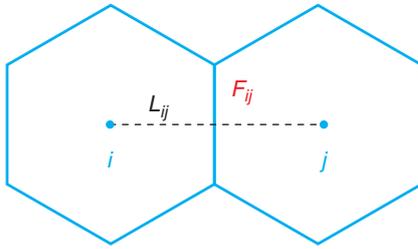


Figure 8
Sketch of adjacent cells of a k-orthogonal grid, in a homogenous reservoir

representation was based on simple kernels. In this paper, we present a methodology using an integral representation of the potential field based on elementary Green's function surfaces that are given analytically. We derive the corresponding analytical kernels for each surface type. The producing fracture surfaces are then discretized and elementary surfaces contributions are distributed over each surface. This leads to a global linear system involving interaction coefficients that has to be solved to insure the boundary condition at the producing surfaces. If the problem at hand can be reduced to 2D (fully perforating vertical fracture for example), an analytical solution to the corresponding system is used, leading to a very fast transmissibility calculations scheme.

Let us start by considering two adjacent cells of a k-orthogonal grid, in a homogenous reservoir (Fig. 8).

The flux between the two cells is classically written as a function of the pressure drop through the use of a constant transmissibility:

$$Q_{ij} = \frac{1}{\mu} T_{ij} (\bar{P}_i - \bar{P}_j)$$

where \bar{P}_i and \bar{P}_j are the average pressures in each cell.

The usual linear assumption on the pressure field leads to the following standard expression for transmissibility:

$$T_{ij} = \frac{k \cdot F_{ij}}{L_{ij}}$$

Above, F_{ij} is the surface of the face between the two cells, and L_{ij} is the distance between the two nodes.

If the pressure field cannot be considered linear in the two cells, another expression has to be found. Let us express the average pressure in each cell (of volume v_i):

$$\bar{P}_i = 1/v_i \cdot \iiint_{C_i} P dv$$

Alternatively, Darcy's law gives:

$$Q_{ij} = \iint_{F_{ij}} \bar{u} \cdot d\vec{S} = \frac{-k}{\mu} \cdot \iint_{F_{ij}} \nabla P \cdot d\vec{S}$$

The transmissibility can hence be expressed as:

$$T_{ij} = -k \frac{\iint_{F_{ij}} \nabla P \cdot d\vec{S}}{1/v_i \cdot \iiint_{C_i} P dv - 1/v_j \cdot \iiint_{C_j} P dv}$$

and the well index is given by:

$$WI = -k \frac{\iint_{F_{iw}} \nabla P \cdot d\vec{S}}{1/v_i \cdot \iiint_{C_i} P dv - P_w + S \cdot \iint_{F_{iw}} \nabla P \cdot d\vec{S}}$$

If the fluid compressibility is further neglected, the pressure field becomes harmonic and can be expressed in terms of Greens function representation. For a source point located at \vec{x}_s , we define a free space function by $G^0(\vec{x}; \vec{x}_s) = \lambda(\vec{x}_s)/\|\vec{x} - \vec{x}_s\|$. Assuming a distribution of source points, the free space function can be integrated to obtain the resulting potential field:

$$\varphi(\vec{x}) = \int_s G^0(\vec{x}; \vec{x}_s) d\vec{x}_s$$

This field being directly proportional to the incompressible pressure field, the formulae for transmissibility and well index can be used directly with the potential in place of the pressure.

3 SOLUTION PROCEDURE

The evaluation of the potential field is done differently depending on the complexity of the problem. If the problem at hand is two-dimensional (fully perforating fractures for example), an analytical solution can be found by conformal mapping techniques. When the problem becomes a three-dimensional one, a numerical evaluation procedure must be used to evaluate the potential function. In this case, the surface is discretized in terms of elementary surfaces and the linearity of the problem at hand allows for direct superposition of individual contribution to obtain the resulting potential field.

Another difficulty arises from the required boundary condition at the producing surfaces: the superposition of singularity distribution with the same strength leads to a solution equivalent to the uniform flux solution, *i.e.* the pressure is not constant along the producing area, on each elementary surface. In order to obtain a constant boundary pressure condition (infinite conductivity fracture for example), a numerical procedure must be involved to compute distribution strength on each elementary surface.

For two-dimensional problems, conformal mapping techniques give closed form solutions for given geometry and boundary conditions. In the present case, we can

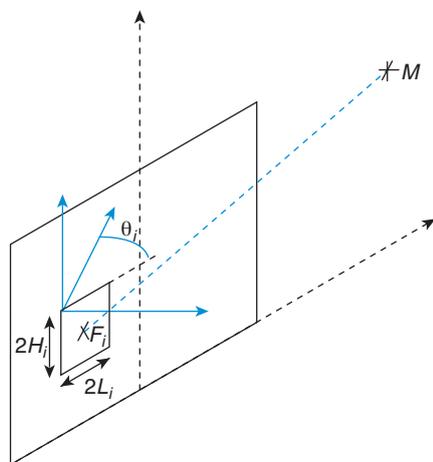


Figure 9
Fracture description.

consider the potential between co-focal ellipses with a degenerate inner ellipse given by:

$$\varphi(x, y) = \ln\left(\frac{a + b}{X_f}\right), \text{ where } X_f \text{ is the half segment}$$

$$\text{length and } \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$b = \sqrt{\frac{1}{2} \left[(x^2 + y^2 - X_f^2) + \sqrt{(x^2 + y^2 - X_f^2)^2 + 4y^2 X_f^2} \right]}$$

$$\text{and } a = \sqrt{X_f^2 - b^2}$$

This solution can be readily used for fully perforating fractures. Note however that in multi-fracture cases the superposition of an array of such solutions would lead to a solution that would not fulfil the constant pressure condition at the fracture faces. In this case, the potential field is built from superposition of local potential fields (no interferences).

For complex three-dimensional problems, such as slanted wells or partially penetrating fractures, the two-dimensional solution cannot be used and closed form three-dimensional solutions are limited to very simple geometries.

The source configuration, composed of either well segments or individual fractures, is subdivided into subsections onto which a constant source distribution is considered (Fig. 9):

We have:

$$\varphi(M) = \sum_i \beta_i \cdot \varphi_u^{rec}(M, F_i, L_i, H_i, \theta_i)$$

where φ_u^{rec} is the panel unitary source solution as given in Appendix. Assuming vertical fractures, the two first images

are accounted for by super-imposing the corresponding solutions:

$$\varphi(M) = \sum_i \beta_i \cdot \left\{ \varphi_u^{rec}(M, F_i, L_i, H_i, \theta_i) + \varphi_u^{rec}(M, F_i^U, L_i, H_i, \theta_i) \right\} + \varphi_u^{rec}(M, F_i^D, L_i, H_i, \theta_i)$$

with F_i^U and F_i^D corresponding to the images of the section center with respect of the upper and lower bound (horizon) respectively.

The determination of the β_i coefficients is done differently depending on the boundary conditions insured at the producing surfaces. If a uniform flux condition is desired, these coefficients are all set to unity resulting in a varying potential along the producing surface. In this case, the segmentation is reduced to a single element and the computation is straightforward. In the more complex case where a constant potential at the surface is required, corresponding to the infinite conductivity condition, the equality of potential between a set of points chosen at the center of each subsection is expressed. This results in a linear system that has to be solved in order to obtain these coefficients.

A similar technique is employed in the case of wells with multiple segments and/or slanted geometry where the unitary panel solution φ_u^{rec} is replaced by unitary segment solution φ_u^{Seg} as given in Appendix.

4 NUMERICAL EXAMPLES

In order to properly assess the validity of the proposed methodology, we consider four examples with fractures in low permeability medium and restrict the simulation to the linear PVT case. In this context, we can compare to analytical solutions computed using Ozkan and Raghavan methodology [8].

Case a) 2D Single Fracture Constant Pressure Production

We first consider a case with a single fracture (see parameters in Tab. 2) which has a complete penetration in a homogeneous layer. In this case, the potential is purely 2D. The well is set to produce at constant bottom-hole pressure for 27 years.

Figure 10 displays the production rate and cumulative production obtained with and without the proposed transmissibility corrections, compared to that obtained from the reference solution. Figure 11 displays the cumulative relative production error with and without transmissibility corrections.

It is clear that the solution obtained without transmissibility corrections leads to an increasing error with time (we extended it to 100 years and obtained 5% error on the standard cumulative). On the contrary, the proposed solution leads to a bounded error, which in this case is always lower than 1% in terms of cumulative production.

TABLE 2
Reservoir and well properties

Reservoir data	
Initial reservoir pressure (psia)	5 000
Net pay (ft)	100
Porosity (%)	10
Permeability (mD)	10^{-3}
Well data	
Number of fractures	1
Fractures half-length X_f (ft)	400
Fractures position	Centered
Fractures penetration	Full
Fractures conductivity F_c (md.ft)	Infinite
Production duration (year)	27

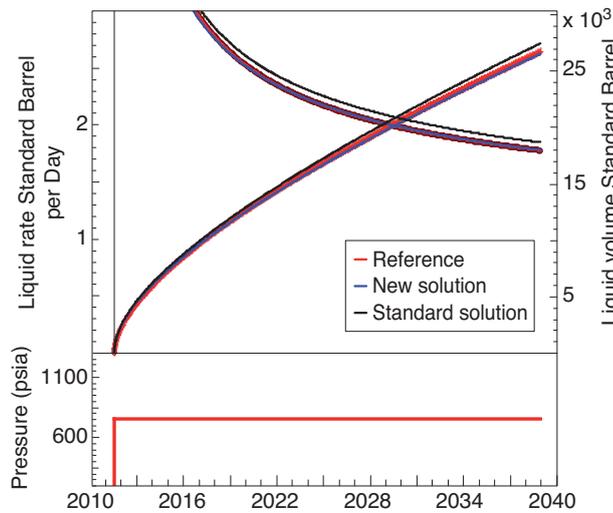


Figure 10
Cumulative production with and without transmissibility corrections *versus* reference solution (red).

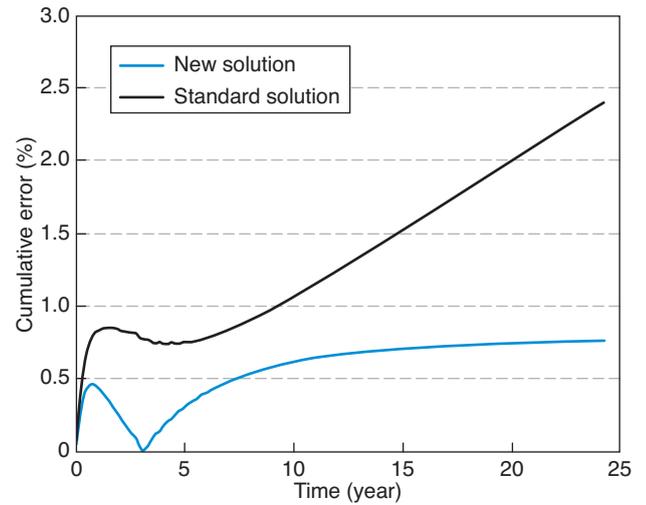


Figure 11
Cumulative production error (relative to reference cumulative) with and without transmissibility corrections *versus* time.

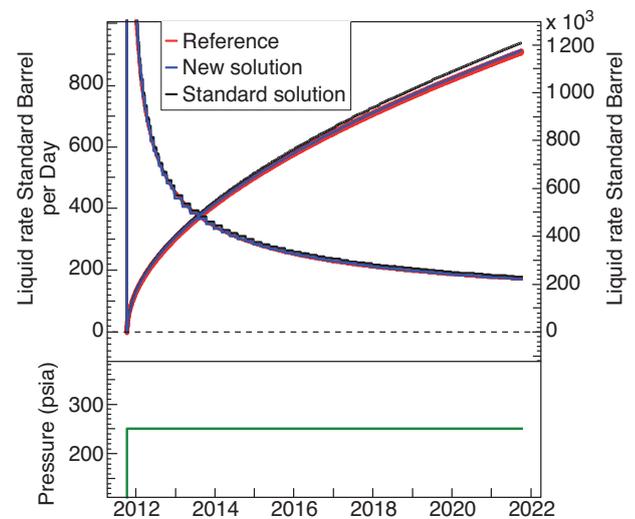


Figure 12
Cumulative production with and without transmissibility corrections *versus* reference solution (thick red line).

Case b) 2D Multi-Fracture Production

Next, we consider again the example of Section 1, summarized in Table 1, with $k = 10^{-4}$ mD. It is a 20 fractures case with complete penetration in a homogeneous layer. In this case, the potential is still 2D.

The well is set to produce at constant bottom-hole pressure for 10 years. Figures 12 and 13 display the cumulative production obtained analytically (“reference”) and numerically, with and without correction.

It is worth mentioning that this case exhibits strong interferences between fractures during production. Even in this context, we see that our solution remains very robust.

Figure 14 displays the cumulative relative production error with and without transmissibility corrections. The solution obtained without transmissibility corrections leads as before to an increasing error with time, reaching 3% after 10 years of production. The proposed solution leads to a bounded error which is in this case is lower than 0.5% in terms of cumulative production.

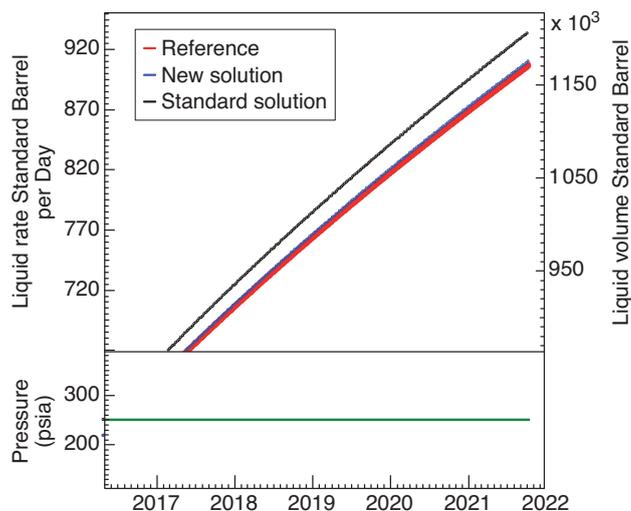


Figure 13

Cumulative production with and without transmissibility corrections *versus* reference solution (thick red line).

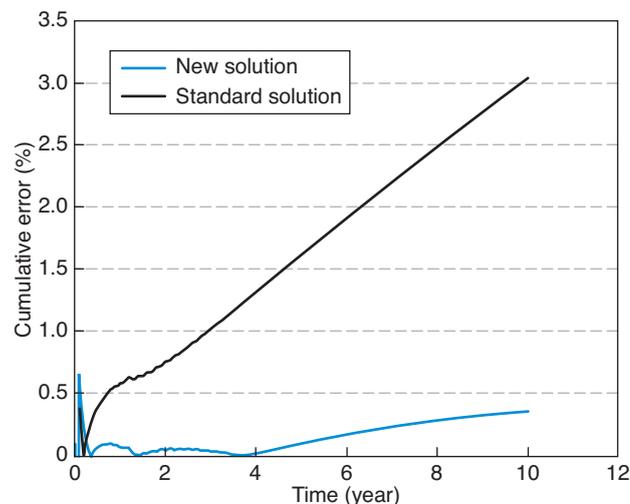


Figure 14

Cumulative production error (relative to reference cumulative) with and without transmissibility corrections *versus* time.

TABLE 3

Reservoir and well properties

Reservoir data	
Initial reservoir pressure (psia)	5 000
Net pay (ft)	100
Porosity (%)	10
Permeability (mD)	10^{-3}
Well data	
Number of fractures	1
Fractures half-length X_f (ft)	400
Fractures position	Centered
Fractures penetration (ft)	40
Fractures top depth (ft)	20
Fractures conductivity F_c (md.ft)	Infinite
Production duration (years)	27

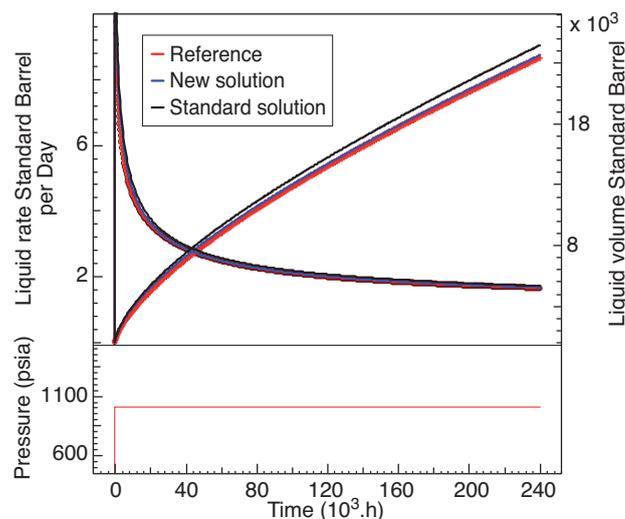


Figure 15

Cumulative production with and without transmissibility corrections *versus* reference solution (thick red line).

Case c) 3D Limited Height Single Fracture Constant Pressure Production

We now consider a 3D problem with a single limited height fracture (*Tab. 3*). Figures 15 and 16 display the production rate and cumulative production as well as the cumulative relative production error with and without transmissibility corrections. As for the 2D problems, the new corrections lead to a clear improvement in this case.

Case d) 3D Limited Height Multi-Fracture Constant Pressure Production

We finally consider a complex 3D problem with a limited height multi-fractures case (*Tab. 4*). Figure 17 displays the truncated numerical grid in this case where the colors correspond to the magnitude of the calculated potential.

Figures 18 and 29 display the production rate and cumulative production as well as the cumulative relative production error

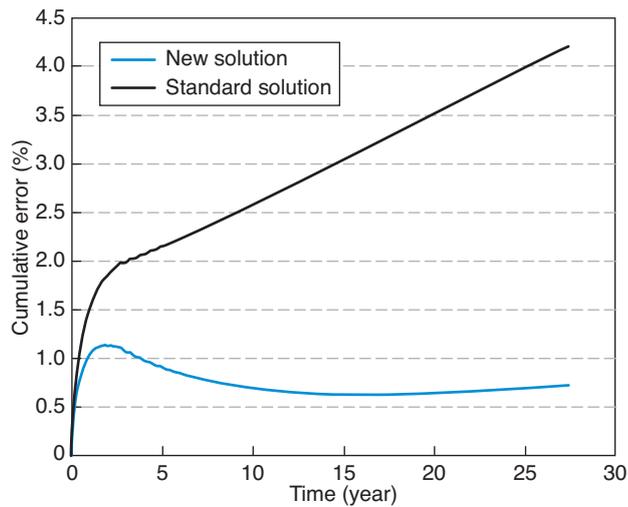


Figure 16
Cumulative production error (relative to final reference cumulative) with and without transmissibility corrections versus time.

with and without transmissibility corrections in this case. As for the previous examples, the new corrections lead to a clear improvement while the standard solution shows an error on the cumulative production which is increasing with time.

5 GRID REFINEMENT CONTROL

Analysing Figure 5, we saw that the extremely low value of the permeability in shale gas context enforces the use of a very fine grid in order to correctly simulate early-time transient

TABLE 4
Reservoir and well properties

Reservoir data	
Initial reservoir pressure (psia)	5 000
Net pay (ft)	100
Porosity (%)	10
Permeability (mD)	10^{-3}
Well data	
Well length (ft)	2 000
Number of fractures	5
Fractures half-length X_f (ft)	400
Fractures position	Centered
Fractures penetration (ft)	40
Fractures top depth (ft)	10
Fractures conductivity F_c (md.ft)	Infinite
Production duration (years)	27
Well completion type	Cased hole

effects. In many cases, however, the early-time behaviour is not essential to the interpretation and a coarser grid may be sufficient, as long as later time scales are correctly captured. This can be properly addressed with numerical near-well upscaling [9, 10].

Using this methodology, the well index and the transmissibility values of the coarse grid are corrected using the results of a steady-state, fine-grid simulation. Figure 20 shows the resulting cumulative obtained for various grid sizes, for the example of Table 1, with $k = 10^{-4}$ mD and linear PVT. We see that the overall process is extremely robust, so

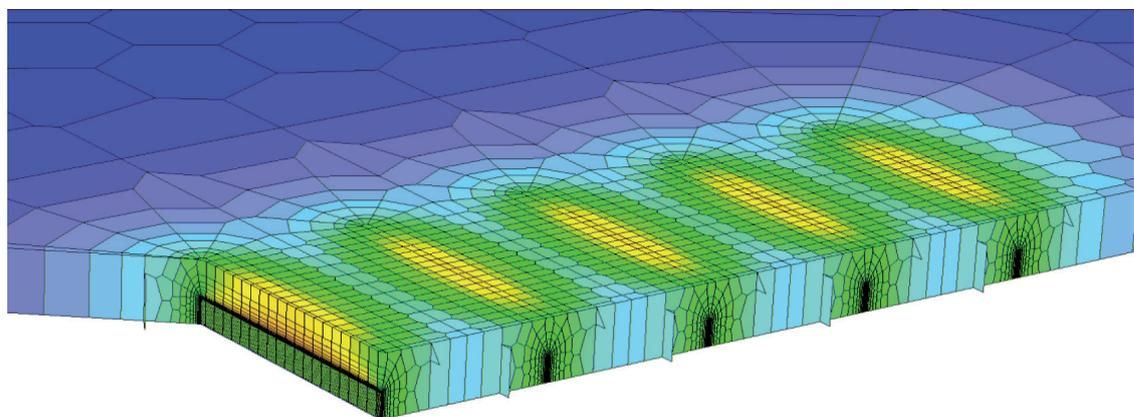


Figure 17
Truncated numerical grid. The colors correspond to the calculated potential.

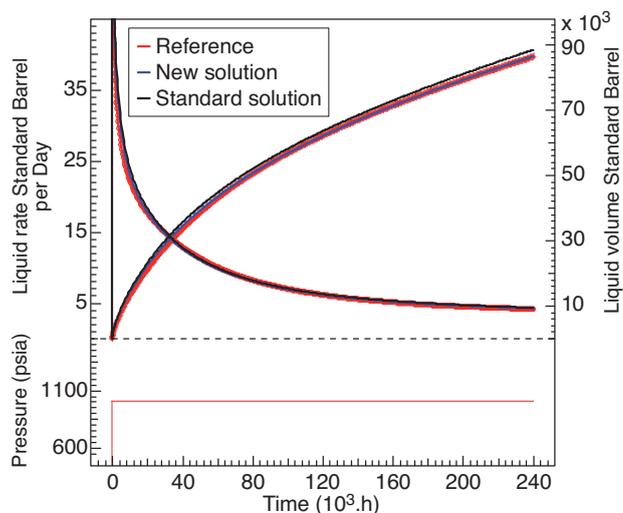


Figure 18

Cumulative production with and without transmissibility corrections *versus* reference solution (thick red line).

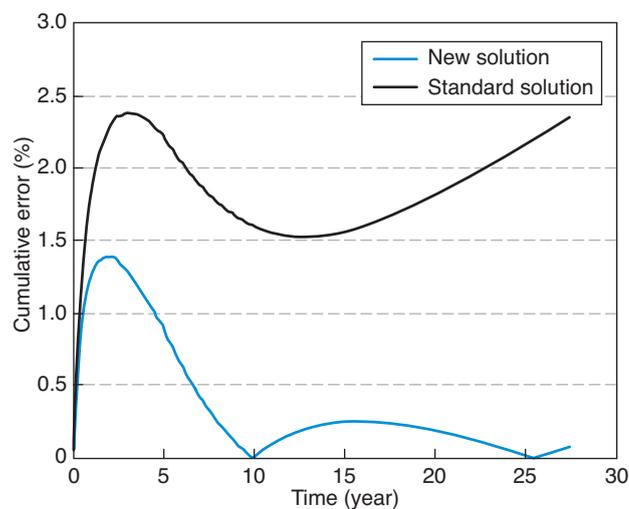


Figure 19

Cumulative production error (relative to final reference cumulative) with and without transmissibility corrections *versus* time.

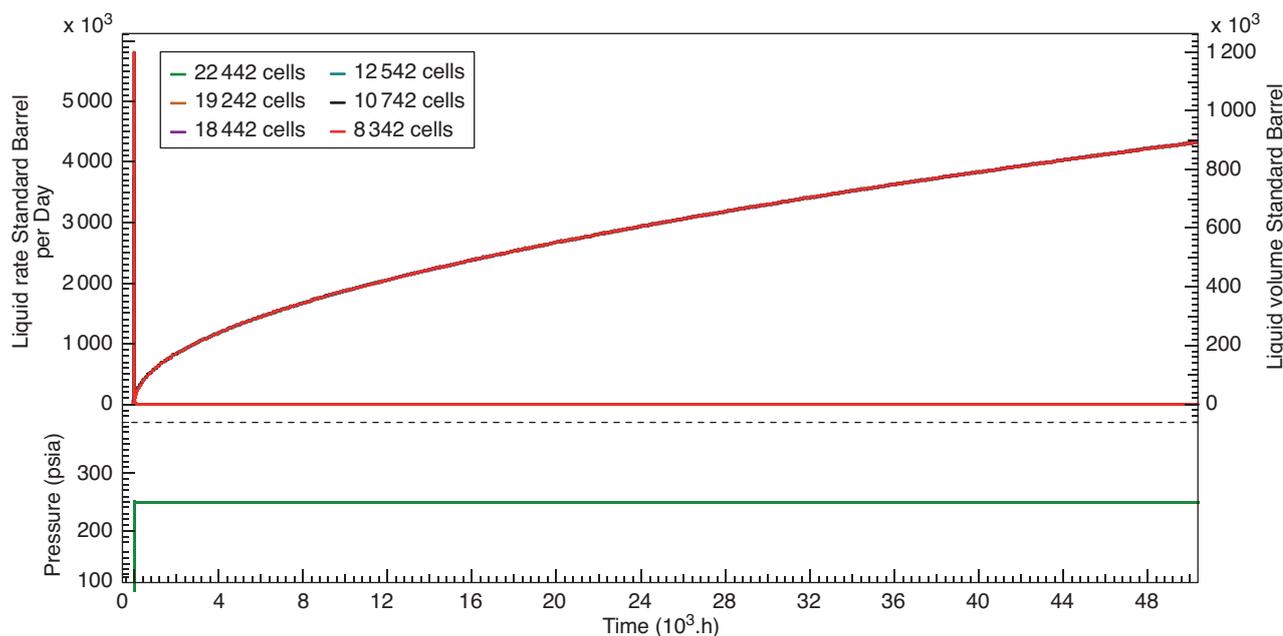


Figure 20

Numerical forecasts for various upscaling levels.

forecasts match perfectly. Note that in this example, the gain on the number of cells is reached only by coarsening the size of the first rings of cells around the fractures (from millimetres to meters scale) without changing the back grid. Note that the excellent match at late-time is not a surprise since the original field was homogeneous. However, the overall procedure also gives very good results at late time, even with near-well heterogeneity.

On the loglog plot (*Fig. 21*), we see that early times are lost after the upscaling process. This is logical: while the grid is coarsened, near-fracture cells become too large to capture transient effects correctly. We see that, however, after a given time which depends on the coarsening level, any curve nicely comes back on the derivative of the finest, reference simulation. What is important to notice here is that even for a mild coarsening level, the correct answer is captured only

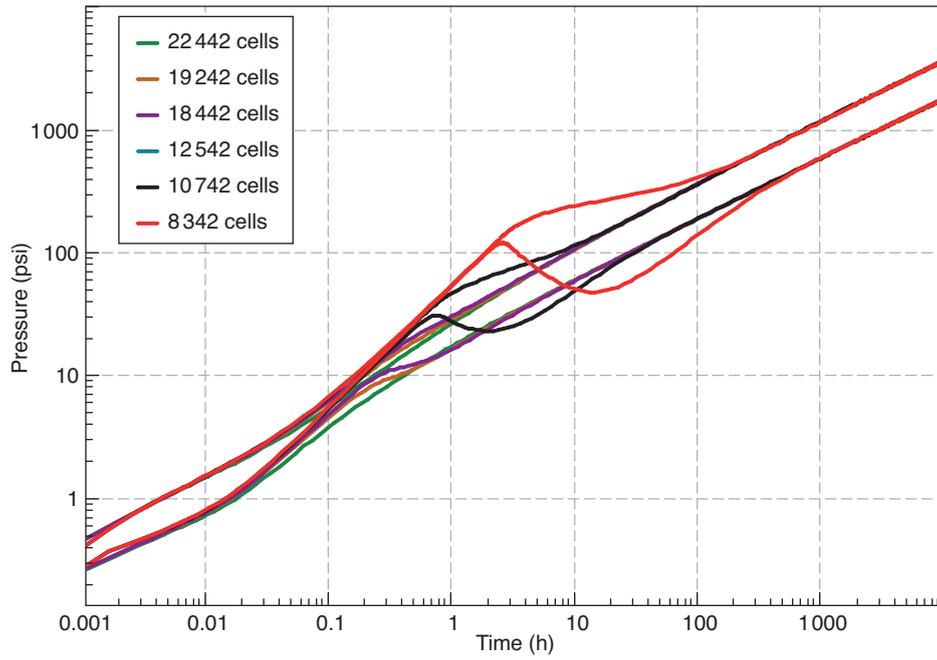


Figure 21
Transient numerical results for various upscaling levels.

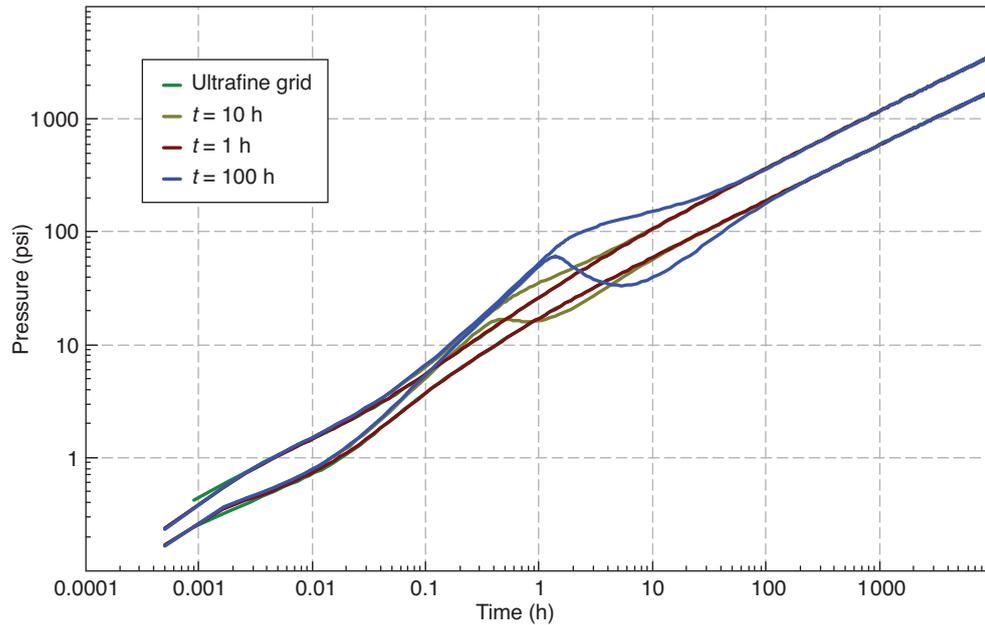


Figure 22
Transient numerical results for various “time resolution” choices, with the automatic grid setting procedure – linear PVT.

after 100 h, because of the very low permeability. For a conventional permeability value, it would only be a matter of minutes even with a larger upscaling level. This raises a practical problem, as one would like to choose the best

coarsening level depending on the desired time resolution of its analysis, *i.e.* without always simulating on the finest grid. In [3], the time scale resolution of a simulation was investigated as a function of the coarsening level. The

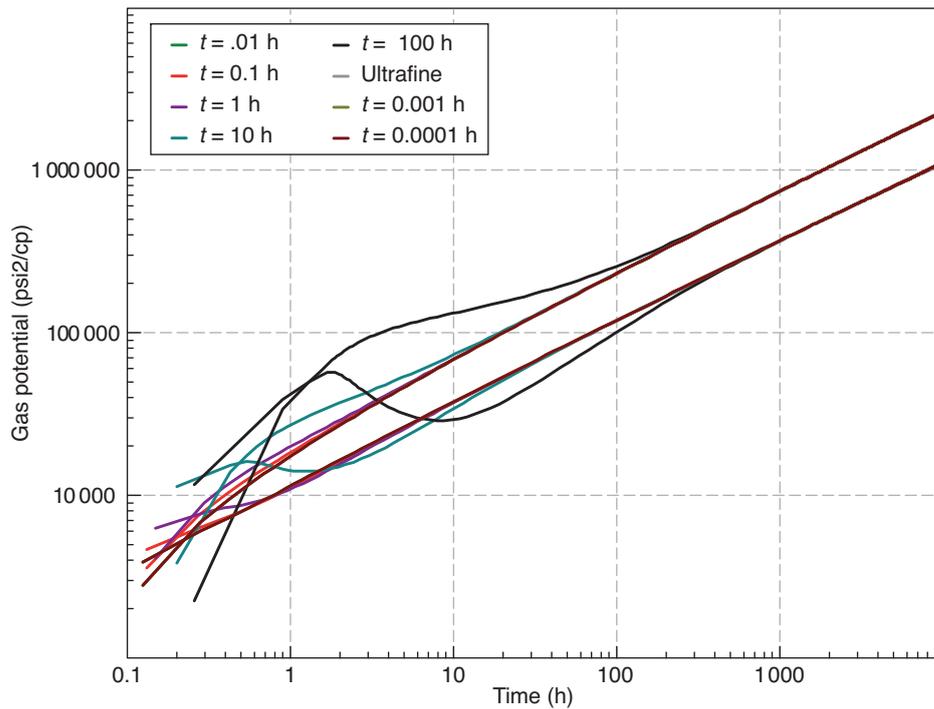


Figure 23

Transient numerical results for various “time resolution” choices, with the automatic grid setting procedure – non-linear gas PVT.

conclusion was that the size of the first grid cells (*i.e.* the cells close to the well or the fractures) may be chosen as a simple function of the investigation radius r , r being evaluated at the desired time resolution Δt :

$$r = \sqrt{\frac{k}{\varphi \cdot \mu \cdot ct} \Delta t}$$

Above, k and φ are the porosity and permeability of the formation, μ is the fluid viscosity and ct is the total compressibility of the system.

We implemented this solution, taking half the investigation radius as the guess for the size of first cells. As can be seen from Figure 22, the results are quite consistent with the expectations.

One can notice that the curve obtained with the 1 h choice for the resolution even matched the ultrafine grid much earlier. This is because we want to ensure the conformity of fine and coarse grids. As a consequence, we do not have full flexibility on the first cell size and the safest choice is taken. In this example, the finest grid contained 22 442 cells, against 13 742 cells for the 1 h resolution grid. The 100 h resolution grid involved 9 542 cells.

These very robust results were obtained because the PVT used was linear. If we use a real, non-linear gas PVT and produce with a large pressure drop (from 5 000 to 250 psi),

the results are still qualitatively acceptable. However, the effective resolution of the simulation is somewhat lower than expected (*Fig. 23*). This can be explained by the fact that the compressibility and viscosity of the fluid change significantly within the space occupied by the first cells. This effect cannot be corrected by the upstream scheme. Hence, the pressure at which compressibility and viscosity are evaluated when deriving the investigation radius should be carefully chosen, as some average values are not sufficient. As shown in Figure 23, the consequences are not dramatic with gas, even with a large pressure drop. When dealing with shale oil, however, one should ensure that the viscosity of the oil does not encounter severe variation within the pressure range corresponding to the first cells, otherwise the automatic refinement may not be suited and even the cumulative may start deviating from the fine simulation.

In this study, we limited our analysis to very simple assumptions regarding the geometry of the fractures. For this reason, we didn't encounter any relevant inaccuracy problem associated with the coexistence of gridblocks of very different volumes (fracture *vs* matrix blocks). This, however, may become more problematic when we extend this work to the simulation of natural networks of fractures [11]. In this case, other simple quality control indicators may have to be derived.

CONCLUSIONS

Specific transmissibility derivations have been developed to address the production of multi-fractured horizontal wells in low-permeability formations. New transmissibilities are shown to improve the coherency between analytical and numerical production cumulatives, compared to the standard, linear case. It should be noticed, however, that these corrections assume high fracture permeability compared to the matrix.

When the matrix permeability is very low, transients last much longer than in the case of conventional permeability. As a consequence, for transient analysis, the size of the grid must be carefully chosen in order to account for the expected effective resolution of the simulation. An automatic grid refinement procedure has been implemented, which depends on the investigation radius at the desired resolution time scale. This process is shown to be quite robust, as long as non-linear PVT effects within the first cell are not too strong.

The proposed solutions are shown to significantly reduce numerical errors linked to the low-permeability, transient context. These initial errors can be shown to be in the same range as the uncertainty due to physical parameters in some real cases [2]. Before history matching real data, such validation hence seems essential to avoid using purely effective physical parameter values in order to simply compensate for strong numerical effects.

ACKNOWLEDGMENT

The authors wish to acknowledge Kappa Engineering for permission to publish this work. Simon Trin and Diego Molinari (Kappa Engineering) are also gratefully acknowledged for early tests of the grid control approach and for their faithful struggle against early versions of the code for transmissibility corrections.

REFERENCES

- 1 Houzé O., Tauzin E., Artus V., Larsen L. (2010) *The Analysis of Dynamic Data in Shale Gas Reservoirs – Part 1*.
- 2 Houzé O., Trin S., Tauzin E. (2010) *The Analysis of Dynamic Data in Shale Gas Reservoirs – Part 2*.
- 3 Molinari D. (2010) Optimization of Near-Well Upscaling in Voronoi Grids, *MSc Thesis*, Politecnico di Torino.
- 4 Lee S.H. (1989) Analysis of Productivity of an Inclined Well and Its Implication for Finite-Difference Reservoir Simulation, *SPE Prod. Eng.* **4**, 2, 173-180.
- 5 Lee S.H., Milliken W.J. (1993) The Productivity Index of an Inclined Well in Finite-Difference Reservoir Simulation, *12th SPE Symposium on Reservoir Simulation*, New Orleans, 28 Feb.-3 March, *SPE* 25247.
- 6 Ding Y. (1996) Modeling of Fracture Wells in Reservoir Simulation, *SPE Annual Technical Conference and Exhibition*, Denver, Colorado, USA, 6-9 Oct., *SPE* 36668.
- 7 Ding Y. (1995) A Generalized 3D Well Model for Reservoir Simulation, *SPE Annual Technical Conference and Exhibition*, Dallas, USA, 22-25 Oct., *SPE* 30724.
- 8 Ozkan E., Raghavan R. (1991) New Solutions for Well-Test Analysis Problems: Part 1 – Analytical Considerations (includes associated papers 28666 and 29213), *SPE Formation Evaluation* **6**, 3, 359-368. *SPE* 18615.
- 9 Ding Y. (1995) Scaling-Up in the Vicinity of Wells in Heterogeneous Field, *13th SPE Symposium on reservoir Simulation*, San Antonio, Texas, 12-15 Feb., *SPE* 29137.
- 10 Mascarenhas O., Durlofsky L. (2000) Coarse Scale Simulation of Horizontal Wells in Heterogeneous Reservoirs, *J. Petrol. Sci. Eng.* **25**, 135-147.
- 11 Landereau P., Noetinger B., Quintard M. (2001) Quasi-steady two-equation models for diffusive transport in fractured porous media: large-scale properties for densely fractured systems, *Adv. Water Resour.* **24**, 863-876.

*Final manuscript received in August 2012
Published online in November 2012*

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APPENDIX

UNITARY SOLUTIONS

3D Segment Source Solution with Linearly Distributed Source

Considering a source distribution of the form $\lambda(s) = \alpha \cdot |s| + \beta$, the potential due to a segment source of length $2L$ is:

$$\varphi(r, z) = \int_{-L}^L \frac{\alpha \cdot |s| + \beta}{\sqrt{(z-s)^2 + r^2}} ds$$

After integration we get:

$$\begin{aligned} \varphi(r, z) = & \alpha \cdot \left(\sqrt{r^2 + (L-z)^2} + \sqrt{r^2 + (L+z)^2} + 2\sqrt{r^2 + z^2} \right) \\ & + \alpha \cdot z \cdot \text{Log} \left(\frac{\left[\sqrt{r^2 + (L-z)^2} + (L-z) \right] \cdot \left[\sqrt{r^2 + (L+z)^2} + (L+z) \right]}{\left(\sqrt{r^2 + z^2} - z \right)^2} \right) \\ & + \beta \cdot \text{Log} \left(\frac{\sqrt{r^2 + (z-L)^2} - (z-L)}{\sqrt{r^2 + (z+L)^2} - (z+L)} \right) \end{aligned}$$

Further considering uniform source distribution, $\lambda(s) = 1$, the solution simplifies to:

$$\varphi_u^{Seg}(M, F, L, \theta, \psi) = \text{Log} \left(\frac{\sqrt{r^2 + (z-L)^2} - (z-L)}{\sqrt{r^2 + (z+L)^2} - (z+L)} \right)$$

In this case one can also express analytically the gradient $\nabla \varphi_u^{Seg}(M, F, L, \theta, \psi)$.

3D Panel Source Solution with Linearly Distributed Source

Considering a source distribution along the panel length of the form $\lambda(s) = \alpha \cdot |s| + \beta$, the potential due to a rectangular source of length $2L$ and height $2H$ is:

$$\begin{aligned} \varphi(r, z, H) = & \delta(r, z, H, h1) - \delta(r, z, H, h0) \\ \delta(r, z, H, h) = & \alpha \frac{h}{2} (g_1 + g_2 - 2g_z) + \text{Log}(2(g_1 + h)) \cdot \left[\frac{\alpha}{2} (R^2 + b_1^2) - b_1 \cdot (\alpha \cdot z + \beta) \right] \\ & + \text{Log}(2(g_2 + h)) \cdot \left[\frac{\alpha}{2} (R^2 + b_2^2) - b_2 \cdot (\alpha \cdot z - \beta) \right] - \text{Log}(2(g_z + h)) \cdot \alpha (R^2 - z^2) \\ & + h \cdot [(\alpha \cdot z + \beta) \cdot \text{Log}(g_1 - b_1) + (\alpha \cdot z - \beta) \cdot \text{Log}(g_2 - b_2) - 2\alpha \cdot z \cdot \text{Log}(g_z - z)] \\ & + R \cdot \left[(\alpha \cdot z + \beta) \cdot \tan^{-1} \left(\frac{h \cdot b_1}{R \cdot g_1} \right) + (\alpha \cdot z - \beta) \cdot \tan^{-1} \left(\frac{h \cdot b_2}{R \cdot g_2} \right) - 2\alpha \cdot z \cdot \tan^{-1} \left(\frac{h \cdot z}{R \cdot g_z} \right) \right] \end{aligned}$$

$$\text{where: } \begin{cases} b_1 = z - L & g_1 = \sqrt{h^2 + b_1^2 + R^2} \\ b_2 = z + L & g_2 = \sqrt{h^2 + b_2^2 + R^2} \\ & g_z = \sqrt{h^2 + z^2 + R^2} \end{cases}$$

Some special cases of interest can be derived from there, the uniform point source distribution, $\lambda(s) = 1$ is readily given in Cartesian coordinates as:

$$\begin{aligned} \varphi_u^{rec}(M, F, L, H, \theta) = & -b_1 \cdot \text{Log} \frac{g_1(z+H) + (z+H)}{g_1(z-H) + (z-H)} + b_2 \cdot \text{Log} \frac{g_2(z+H) + (z+H)}{g_2(z-H) + (z-H)} \\ & + (z+H) \cdot \text{Log} \frac{g_1(z+H) - b_1}{g_2(z+H) - b_2} - (z-H) \cdot \text{Log} \frac{g_1(z-H) - b_1}{g_2(z-H) - b_2} \\ & + R \cdot \left[\tan^{-1} \left(\frac{b_1(z+H)}{Rg_1(z+H)} \right) - \tan^{-1} \left(\frac{b_1(z-H)}{Rg_1(z-H)} \right) - \tan^{-1} \left(\frac{b_2(z+H)}{Rg_2(z+H)} \right) + \tan^{-1} \left(\frac{b_2(z-H)}{Rg_2(z-H)} \right) \right] \end{aligned}$$

$$\begin{aligned} b_1 = x \frac{dx}{L} + y \frac{dy}{L} - L & \quad g_1(h) = \sqrt{h^2 + (x+dx)^2 + (y+dy)^2} & \quad R = y \frac{dx}{L} - x \frac{dy}{L} \\ b_2 = x \frac{dx}{L} + y \frac{dy}{L} + L & \quad g_2(h) = \sqrt{h^2 + (x-dx)^2 + (y-dy)^2} & \quad dx = L \cos \theta, dy = L \sin \theta \end{aligned}$$

$[x, y, z] = M - F$, F the panel centre.