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Economics and the Refinery's CO_2 Emissions Allocation Problem

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Résumé — Allocation des émissions de CO_2 d'une raffinerie : solutions suggérées par la théorie économique — L'établissement d'un marché européen de permis d'émissions de CO_2 a conduit les compagnies pétrolières à prendre en compte le coût de ces émissions dans les modèles de programmation linéaire utilisés pour gérer leurs raffineries. Ces modèles permettent de déterminer la contribution marginale de chaque produit fini aux émissions de CO_2 de la raffinerie. Babusiaux (*Oil. Gas Sci. Technol.*, 58, 2003, 685-692) a montré que, sous certaines hypothèses, cette contribution marginale constitue une clé de répartition des émissions de la raffinerie particulièrement pertinente, pouvant alors être utilisée dans le cadre d'analyses en cycle de vie « du puits à la roue ». Il faut pour cela que les contraintes de demande en produits finis soient, à l'optimum, les seules contraintes saturées à second membre non nul. Cette hypothèse n'est certainement pas vérifiée lorsqu'est employé un modèle de court terme, dans lequel les capacités sont fixées. Allouer les émissions de carbone sur une base marginale surévalue (ou sous-évalue) alors le volume total des émissions. La théorie économique suggère deux solutions possibles à ce problème : adapter la formule d'Aumann-Shapley (*Values of non-atomic games,* 1974, Princeton University Press) ou celle de Ramsey-Boîteux (*Econ. J.*, 37, 1927, 47-61 ; *J. Econ. Theory,* 3, 1971, 219-240). Celles-ci sont comparées. Un argument, basé sur la détermination d'une taxe environnementale à laquelle des produits importés devraient être assujettis, joue en faveur d'un système de prix de type Ramsey-Boîteux.

Abstract — Economics and the Refinery's CO_2 Emissions Allocation Problem — *The establishment of a market for CO_2 emission rights in Europe leads oil-refining companies to add a cost associated with carbon emissions to the objective function of linear programming models used to manage refineries. These models may be used to compute the marginal contribution of each finished product to the CO_2 emissions of the refinery. Babusiaux (*Oil. Gas Sci. Technol.*, 58, 2003, 685-692) has shown that, under some conditions, this marginal contribution is a relevant means of allocating the carbon emissions of the refinery. Thus, it can be used in a well-to-wheel Life Cycle Assessment. In fact, this result holds if the demand equations are the only binding constraints with a non-zero right-hand side coefficient. This is not the case for short-run models with fixed capacity. Then, allocating CO_2 emissions on a marginal basis tends to overvalue (or undervalue) the total volume of emissions. In order to extend the existing methodology, we discuss two distinct solutions to this problem, inspired by economic theory: adapting either the Aumann-Shapley cost sharing method (*Values of non-atomic games, *1974, Princeton University Press) or the Ramsey pricing formula (*Econ. J., *37, 1927, 47-61;* J. Econ. Theory*, 3, 1971, 219-240). We compare these two solutions, with a strong argument in favour of Ramsey prices, based on the determination of the optimal environmental tax rate to which imported finished products should be subject.*

INTRODUCTION

In Europe, the refining sector is confronted with an increase in the market share of diesel fuel, to the detriment of gasoline, due to the dieselization of the fleet of cars. In this context, the public authorities of certain countries must study the benefit of perpetuating a fiscal policy that induces a tax differential between gasoline and diesel. In order to make a decision, these public authorities must consider the externalities attributable to these two fuels, such as emissions of greenhouse gases like CO_2. Furthermore, the recent establishment in Europe of an emission permits market, which includes the petroleum refining sites, can lead the authorities to impose taxes on products imported from countries in which carbon emissions are not taken into account.

For the purposes of making these decisions, life-cycle assessments (LCAs) conducted "well-to-wheel" provide particularly useful information. Performing these assessments rigorously requires the emissions of the refineries to be allocated to the various products coming from them. A petroleum refinery is a group of interdependent refining plants, causing these various products to be produced simultaneously. Consequently, the number of possibilities for allocating the refinery's carbon emissions to these products is infinite.

The allocation methods generally used in life-cycle assessment, such as the method recommended by Wang *et al.* (2004), are not based on marginal analysis. However, it is the marginal approach that is recommended by economic theory for decision-making.

The establishment of an emission permits market in Europe drives refiners to include a cost associated with the emissions in the objective function of the linear programs used to manage their refineries. These refineries thus minimize a scalar function, including the purchase or sale of emission permits, under the constraint of satisfying an exogenous demand for products. By introducing the concept of elementary function[1], Babusiaux (2003) shows that it is possible to determine the marginal contribution of each product to the refinery's total emissions. Under certain conditions, this marginal contribution has an average-contribution structure. In certain cases, the refinery's CO_2 emissions can therefore be allocated on a marginal basis.

They cannot, however, be allocated on a marginal basis when the demand constraints are not the only binding constraints with a non-zero right-hand side coefficient. Short-term models in which the various refining plants' capacities are fixed are of particular concern. As Tehrani Nejad (2006) noted, allocating on a marginal basis can then lead to a great over-estimation (or under-estimation) of the refinery's emissions. We searched for a solution to the problem raised by the allocation of emissions in this type of situation.

1 We here adopt the term used by Babusiaux (2003) and Pierru and Babusiaux (2004).

Because the main purpose of the life-cycle assessments in question is economic decision-making, particular with regards to taxation policy, we sought a solution to the allocation problem studied from economic theory. A review of the literature led us to study two solutions, each of which involved adapting an existing method. Both make use of the elementary dual variable concept associated with the part of the objective function representing the cost of carbon emissions.

The first solution considered consists of adapting the Aumann-Shapley formula, derived from cooperative non-atomic game theory. This solution was studied in detail by Pierru (2006) and will be summarized here.

The second proposed solution consists of adapting a Ramsey-Boîteux price system, which indicates the optimal difference between prices and marginal costs under certain conditions. Ramsey prices have been derived in various fields of economic literature: optimal taxation theory, second best pricing for public enterprises, sustainability of a multi-product monopoly, etc.

We first briefly review the methodology proposed by Babusiaux (2003) for allocating a petroleum refinery's CO_2 emissions and its limitations. We then present both methods and their adaptation in the context of the problem studied. We show that they broaden the existing methodology and discuss their relative advantages and disadvantages.

1 EXISTING METHODOLOGY AND LIMITATIONS

We assume that the refinery's objective is either to maximize a profit or to meet a given demand for finished products at minimum cost. Both formulations are equivalent if the possibility of buying and selling finished products (imports and exports) is included in the second one. In this paper, we shall consider a problem of cost minimization, with exports (production in excess of fixed demand) taken as negative costs.

We will let $b=(b_1, b_2,...,b_m)$ denote the vector representing the quantities demanded for m finished products.

In a refining program, the main endogenous variables are the flows of crude oil to be processed, of intermediate products and finished products. In addition to the demand constraints for finished products, these models take into account three main types of constraints:

- material balance equations, which express the equality between an available quantity of a given intermediate product and the quantities used for the different possible destinations of this product;
- quality constraints, which express each finished product's obligation to meet legal specifications;
- capacity constraints, which reflect the capacity limitations of existing units (in the long-run models used to analyze investment decisions, the capacities of units to be built are considered as variables).

The objective function to minimize, subject to constraints, is the sum of two "elementary" functions:

– the operating cost (plus the investment cost of units to be built in the long-run models used to analyze investment decisions);
– the cost associated with CO_2 emissions (assumed to be equal to the quantities released times the price of an emission permit),

where C is the refiner's cost function, for a given vector b of demand for finished products, at the optimum, the objective function takes the value $C(b)$. The function C is piecewise linear.

Let $C_1(b)$ denote the value taken by the elementary function representing the operating cost and $C_2(b)$ the value taken by the elementary function representing the cost associated with CO_2 emissions. We have:

$$C(b) = C_1(b) + C_2(b)$$

In long-run models, demand constraints are often the only constraints with a non-zero right-hand side coefficient. In this case, if the optimal basic solution is nondegenerate, the sum of the products of every marginal cost by the corresponding quantity demanded is equal to the total cost:

$$\sum_{i=1}^{m} b_i \frac{\partial C}{\partial b_i}(b) = C(b)$$

The marginal cost is then a relevant means of allocating the refinery's cost among the finished products. Babusiaux (2003) - Pierru and Babusiaux (2004) for a more formal presentation of this result - has shown that this property was true for each elementary function:

$$\sum_{i=1}^{m} b_i \frac{\partial C_1}{\partial b_i}(b) = C_1(b) \qquad \sum_{i=1}^{m} b_i \frac{\partial C_2}{\partial b_i}(b) = C_2(b)$$

$\frac{\partial C_1}{\partial b_i}(b)$ and $\frac{\partial C_2}{\partial b_i}(b)$ are called "elementary dual variables". It is thus possible to allocate the cost associated with the refinery's emissions to the various finished products, using the corresponding elementary dual variable $\frac{\partial C_2}{\partial b_i}(b)$. The ratio of $\frac{\partial C_2}{\partial b_i}(b)$ to the emission permit price gives the marginal contribution of product i to the CO_2 emissions of the refinery. The two following requirements are met:

– emissions are allocated on the marginal basis, thus facilitating decision-making (in accordance with economic theory);
– the total quantity of the refinery's emissions is allocated to the various products, which is consistent with an LCA-based approach.

As already mentioned, there are nonetheless situations where this method does not satisfy the second requirement. When demand constraints are not the only binding constraints with a non-zero right-hand side coefficient, it is no longer possible to allocate costs (or emissions) on the marginal basis. Thus, in short-run models, the capacity constraints of existing units have a right-hand side coefficient other than zero. These models can also take into account the availability constraints of certain types of crude oil. It is therefore necessary to elaborate a more general approach, which can best meet the two preceding requirements. We propose using either the Aumann-Shapley cost-sharing method or a Ramsey-type price system. To solve the problem studied here, both methods, inspired by economic theory, need to be adapted.

2 SOLUTIONS FROM ECONOMIC THEORY

2.1 Aumann-Shapley Method

This cost-sharing method has been proposed by Aumann and Shapley (1974). Let us denote $C(b)$ as the cost function of the demand vector b. Under suitable differentiability assumptions, the per-unit cost share (also called "A-S price") imputed to product i with this method, denoted $s_i(b,C)$, is then:

$$s_i(b,C) = \int_0^1 \frac{\partial C}{\partial b_i}(\lambda b)\, d\lambda \qquad (1)$$

The per-unit cost share imputed to product i is the integral of the marginal cost of product i along the ray to b. The per-unit cost shares thus defined allow us to allocate the total cost:

$$\sum_{i=1}^{m} b_i s_i(b,C) = C(b)$$

As here we consider a cost function C which is piecewise linear, the A-S price vector is a sum of the gradients of the linear "pieces" of C along the ray to b, where each of these is weighted by the normalized length of the subinterval in which C has a constant gradient. In other words, the formula (1) is a sum of areas of rectangles, with as many rectangles as there are basic solutions, successively determined along the line from the origin to b.

If there are $n-1$ successive basis changes, where $\lambda_k b$ denotes the output value at which the k-th basis change occurs, we have:

$$s_i(b,C) = \sum_{k=0}^{n-1} (\lambda_{k+1} - \lambda_k) \frac{\partial C}{\partial b_i}(\lambda_k b)$$

where:

$\lambda_0 = 0, \lambda_n = 1, \frac{\partial C}{\partial b_i}\left(\lambda_k b\right)$, ($k$ = 0, 1,..., n–1), stands for the marginal cost of product i associated with the (k+1)-th basis determined.

Our objective here is not to determine an average cost per finished product, but to calculate the contribution of each product to the refinery's total CO_2 emissions. Emission costs must first be allocated to the various products. To that end, we use the breakdown of the objective function into elementary functions suggested by Babusiaux (2003). The basic solutions successively determined when demand moves along the line from the origin to b are found by minimizing the total cost (including operating costs and costs associated with CO_2 emissions). On the other hand, for each of these basic solutions, only the elementary dual variable $\frac{\partial C_2}{\partial b_i}\left(\lambda_k b\right)$ associated with the cost of the emissions has to be taken into account in the formula above. Where $s_i(b,C_2)$ denotes the contribution (per-unit) of the product i to the cost associated with carbon emissions:

$$s_i\left(b, C_2\right) = \sum_{k=0}^{n-1}\left(\lambda_{k+1} - \lambda_k\right)\frac{\partial C_2}{\partial b_i}\left(\lambda_k b\right)$$

Pierru (2006) calls $s_i(b,C_2)$ the "elementary A-S cost associated with CO_2 emissions" as its calculation is based on the elementary dual variable concept $\frac{\partial C_2}{\partial b_i}\left(\lambda_k b\right)$. The contribution of the product i to the refinery's emissions is obtained by dividing $s_i(b,C_2)$ by the price of the emission permit.

2.2 Ramsey Pricing

Issues in various economic fields (theory of taxation, analysis of public utility regulation, sustainability of multiproduct natural monopoly) lead to consider Ramsey prices.

The problem initially considered - commonly referred to as the Ramsey problem after the solution proposed by Ramsey (1927) - is that of the optimal configuration of commodity tax rates. The simplest version of the Ramsey problem can be studied with a static model with a representative consumer. The government's objective is to raise a given amount of revenue. The Ramsey's solution to this problem states that the optimal set of commodity taxes leads to an equal percentage reduction in the (compensated) demands for all goods. Rather than changing each price by an equal percentage (uniform taxation), the optimal tax system implies an equal percentage change in the quantities of each good. If demands for different goods are unrelated (cross-elasticities equal zero), then the Ramsey rule simplifies to the "inverse elasticity rule" which states that each tax rate should be inversely proportional to the elasticity of demand for the good considered.

Boiteux (1971) considers a regulated multiproduct monopoly which, because of scale economies, would suffer losses if it were to set the prices of its products equal to the corresponding marginal costs. In theory, the social optimum is obtained by setting prices equal to marginal costs (provided that certain second-order conditions are satisfied). If the public authorities impose that the monopoly's makes a zero-profit, then the constrained welfare optimum (second best) yields a Ramsey pricing system.

Baumol *et al.* (1977) examines the case of a natural multiproduct monopoly using a productive technique (available to other firms) whose cost function exhibits sufficiently strong cost advantages. An entry cost is assumed to face new firms in the market. These authors show that the setting of Ramsey prices by the monopoly is sufficient to prevent the entry of new competitors on the markets of one or several of its products (*i.e.* is sufficient to guarantee the sustainability of the monopoly). Thus, as the authors say: "the same invisible hand that guarantees welfare-optimal pricing under perfect competition, may guide the farsighted monopolist, seeking protection from entry, to the Ramsey welfare optimum".

The simplest way to adapt the Ramsey pricing formula consists in a constrained maximisation of the aggregated consumers and refiners surplus, as presented by Boiteux (1971). Cross-elasticities of demand are assumed to be zero. Let $p_i(b_i)$ denote the inverse demand function for every product i. Producing b generates the following surplus:

$$\sum_{i=1}^{m}\int_0^{b_i} p_i\left(u_i\right)du_i - C\left(b\right)$$

Let us now define the constraint. Here, for each product i, the difference between its price $p_i(b_i)$ and its marginal cost $\frac{\partial C}{\partial b_i}\left(b\right)$ is fully attributed to CO_2 emissions cost. Consequently, its "modified" cost associated with CO_2 emissions is equal to that difference plus the elementary dual variable $\frac{\partial C_2}{\partial b_i}\left(b\right)$. The constraint is therefore: multiplying for each finished product its produced quantity by this modified cost, and adding the resulting figures for all products, gives the total cost of CO_2 emitted by the refinery. Thus, mathematically the constraint is written:

$$\sum_{i=1}^{m} b_i\left(p_i\left(b_i\right) - \frac{\partial C_1}{\partial b_i}\left(b\right)\right) = C_2\left(b\right)$$

We are therefore looking for the optimum of the following program:

$$\underset{b}{Max} \quad \sum_{i=1}^{m} \int_0^{b_i} p_i(u_i)\,du_i - C(b)$$

$$s.t. \quad \sum_{i=1}^{m} b_i \left(p_i(b_i) - \frac{\partial C_1}{\partial b_i}(b) \right) = C_2(b)$$

As the refiner's cost function is piecewise linear, if the optimal basic solution is not degenerate (as is assumed here), we have: $\frac{\partial^2 C_1}{\partial b_j \partial b_i}(b) = 0 \quad \forall (i,j)$. Let λ denote the Lagrange multiplier associated with the constraint. At the optimum, we have for every product *i*:

$$p_i(b_i) - \frac{\partial C}{\partial b_i}(b) - \lambda \times$$

$$\left(p_i(b_i) - \frac{\partial C_1}{\partial b_i}(b) + b_i \frac{dp_i}{db_i}(b_i) - \frac{\partial C_2}{\partial b_i}(b) \right) = 0$$

$$i = 1, 2, \ldots, m$$

Which gives:

$$p_i(b_i) - \frac{\partial C}{\partial b_i}(b) = \frac{\lambda}{1-\lambda} b_i \frac{dp_i}{db_i}(b_i)$$

We obtain the following pricing formula:

$$\frac{p_i(b_i) - \frac{\partial C}{\partial b_i}(b)}{p_i(b_i)} = \frac{k}{\eta_i}$$

where $\eta_i = (p_i / b_i)(db_i / dp_i)$ is the elasticity of demand for product *i* with respect to changes in its price. $k = \frac{\lambda}{1-\lambda}$ is a constant (of proportionality) chosen as required to satisfy the constraint.

As in Babusiaux and Pierru (2007), we obtain a pricing system consistent with the inverse elasticity rule: the relative divergence between price and marginal cost, caused by the additional[2] CO_2 emissions cost, is inversely proportional to the elasticity of demand for the product considered.

3 CONCLUSIONS: RELATIVE ADVANTAGES AND DISADVANTAGES OF METHODS

First, both approaches generalize the method proposed by Babusiaux (2003). When demand constraints are the only constraints with a non-zero right-hand side coefficient, the refiner's cost function is a homogenous function (of degree 1). The use of the A-S method then results in allocating to the various products a contribution equal to their marginal contribution. Ramsey prices are equal to marginal costs (constant of proportionality *k* equal to zero), which leads to the same result.

The A-S method satisfies a marginality property (the cost share imputed to product *i* depends only on the marginal cost function with respect to product *i*), whereas Ramsey prices represent an optimal departure from marginal costs. However, to compute Ramsey prices requires information on demand (price elasticity of demand, at the very least), which the A-S method does not.

As mentioned by Pierru (2006), the elementary A-S costs are calculated in the same way for all products (with the same coefficients weighting the successively determined marginal costs). Furthermore, if the objective function also includes the cost associated with SO_2 emissions, then the calculation of the contribution of each product to the refinery's sulphur emissions would be entirely consistent with the calculation of their contribution to carbon emissions.

Tehrani Nejad (2006) highlights the unpredictable behaviour of the marginal contribution of a given product to the refinery's emissions. As the demand for the product increases gradually, its marginal contribution may change (increase or decrease) abruptly, unlike its marginal cost. This observation makes the use of marginal contributions as part of an LCA delicate. In certain situations, the use of a short-term model, associated with the Aumann-Shapley method, might then present a paradoxical virtue – one that reduces the obtained allocation's sensitivity to the demand conditions – because of the calculation method (weighted average of successive marginal contributions).

To obtain proper data for LCAs requires allocating the carbon emissions of a refinery to the various finished products. These LCAs provide useful information to the public authorities in decision-making, related to, for example, determining tax rates on imported products. By "imported products" we mean finished products imported from countries where CO_2 emissions have no impact on refiners' decisions or pricing policy. Clearly, Ramsey prices fulfil this objective, as the optimal tax rates are equal to the difference between prices and marginal costs. As the determination of these tax rates and the allocation of CO_2 emissions are two facets of the same problem, this is a strong argument in favour of the use of Ramsey prices. It must be noted that the inverse elasticity rule allocates a high tax rate (*i.e.* a high additional environmental cost) to products for which demand is inelastic (since changing their price does not create much economic distortion in the demand). Conversely, lower tax rates are set on price-elastic products, as small price changes may create large distortions. In France, the proportional increase (or decrease) of the allocated environmental burden would be higher for road gas oil than for gasoline.

2 We use the term "additional" here although it involves a negative quantity when allocating on a marginal basis leads to over-estimating the refinery's total emissions.

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