

Nonlinear Internal Model Control of Diesel Air Systems

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Résumé — Modèle interne non-linéaire pour le contrôle des systèmes d'air dans les moteurs Diesel

Avec la complexité croissante des groupes de moto propulsion, les méthodes heuristiques de synthèse de systèmes de régulation à calibration expérimentale ne permettent plus d'assurer un niveau de performance satisfaisant. Cet article introduit un nouveau type de régulateur basé sur un modèle et permettant de répondre aux demandes industrielles du domaine automobile. Le problème traité concerne la régulation de pression de suralimentation d'un moteur diesel équipé d'une turbine à géométrie variable. Le système étant essentiellement non linéaire, une approche robuste non linéaire est utilisée. La commande est fondée sur une structure combinant une structure de commande à modèle interne et l'approche par platitude pour le calcul d'une commande en boucle ouverte. L'idée principale de la commande à modèle interne réside dans l'utilisation d'un modèle du procédé dans la boucle de rétroaction. Si le modèle représente parfaitement le procédé et en l'absence de perturbations, la commande à modèle interne génère une commande en boucle ouverte. D'autre part, les systèmes plats sont caractérisés par le fait que leur entrée peut être exprimée explicitement à l'aide de variables d'états internes du système, facilitant la synthèse de commandes en boucle ouverte. Dans cet article, une commande en boucle ouverte par platitude est employée dans une structure de commande à modèle interne. De plus, la méthode proposée permet de prendre en compte explicitement des contraintes sur le signal de commande généré. Il est montré que cette nouvelle approche constitue une méthode efficace de synthèse de commande pour une classe de systèmes non linéaires et assure des propriétés de robustesse et de suivi de trajectoire à erreur statique nulle. Des résultats de simulation et d'expérimentation en banc d'essai démontrent la faisabilité de cette nouvelle approche affichant des performances très prometteuses.

Abstract — Nonlinear Internal Model Control of Diesel Air Systems — As a result of the increased complexity of today's power trains, the traditional ways of designing engine control systems essentially through ad hoc methods and experimental tuning will no longer provide the desired level of performance. In this paper, a novel model-based controller is described which accommodates many of today's demands on controller development of the automotive industry. The control problem treated here is a boost pressure control of a turbocharged diesel engine with a variable nozzle turbine (VNT). Since the system is essentially nonlinear, a robust nonlinear controller is used. The tracking problem is treated by a control method which combines the Internal Model Control (IMC) structure with the flatness-based approach to design feedforward controllers. The main idea of IMC is to include the model of the plant into the feedback controller. If the model perfectly represents the plant and no disturbances occur, the IMC structure degenerates to a pure feedforward control. Flat systems are characterized by the fact that

their input can be expressed explicitly in terms of internal system dynamics which results in a simple method for designing a feedforward controller. In order to extend this concept to nonlinear systems, a flatness-based feedforward controller is proposed as IMC controller. Furthermore, the introduced method allows to explicitly consider input constraints. It is shown that this new concept provides an efficient controller design for certain nonlinear systems and ensures robustness and offset-free tracking. Simulation and testbed results of a controlled air system of a turbocharged diesel engine demonstrate the feasibility of this control scheme which results in impressive control performance.

INTRODUCTION

In the automotive industry, the need for efficient ways of control system design is rapidly gaining importance because of the growing complexity of new technologies, which results from a rising number of demands on the closed-loop performance, the existence of large plant uncertainties and severe nonlinearities, as well as the increasing number of control variables (more measurement points and inputs).

Hence, systematic ways for designing controllers have to be introduced that ensure sufficient robustness of the closed-loop system and deal with the nonlinearities and input saturations of the plant. However, such control designs must fulfill specific requirements of the automotive industry like low cpu and memory usage and reusability. Additionally, in order to avoid several re-designs, dedicated parameters which influence the control performance should be provided. Thus, controller design and its calibration can be performed by different engineers.

A control design which fulfills these demands is the concept of flatness-based IMC. This concept is introduced and used to control the boost pressure of a turbocharged diesel engine.

This paper is organized as follows: in Section 1 the principle and the properties of flatness-based IMC are briefly recalled. Section 2 presents the main result of this paper: an extension of the flatness-based IMC structure to respect input constraints of the plant. The results are applied to the example system of a turbocharged diesel engine air system in Section 3. Simulation results of the controlled air system, demonstrating the performance of the controller are provided in Section 4, while Section 5 gives additional testbed results.

1 FLATNESS-BASED INTERNAL MODEL CONTROL

This section introduces the combination of two known control approaches: Internal Model Control (IMC) [1] and flatness-based feedforward control [2]. The necessary results from [3] are briefly reviewed and important robust closed-loop properties like zero steady-state offset and stability which also hold in the case of nonlinear systems are recalled. For an more in-depth review of the principles of nonlinear IMC, the reader is referred to [3].

Once a flatness-based IMC has been designed, its internal model parametrization can be altered without sacrificing closed-loop properties. Thus, an IMC can easily be ported to different engine configurations. This way the separation of control algorithm design and calibration is taken into account and the calibration can be performed by non control engineers.

1.1 Internal Model Control

This section gives a survey of the IMC concept and its main structural properties.

1.1.1 Mathematical Background

The plant model $\tilde{\Sigma}$ is defined for $t \in [0, \infty)$ by

$$\tilde{\Sigma}: \dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), u(t)), \quad \mathbf{x}(0) = \mathbf{x}_0, \quad \mathbf{x} \in \mathcal{X} \quad (1a)$$

$$\tilde{\mathbf{y}}(t) = h(\mathbf{x}(t), u(t)), \quad u \in \mathcal{U}, \quad \tilde{\mathbf{y}} \in \tilde{\mathcal{Y}} \quad (1b)$$

with respect to a nonempty set \mathcal{X} (state function-space), a nonempty set \mathcal{U} (input function-space) and a nonempty set $\tilde{\mathcal{Y}}$ (output function-space). The sets \mathcal{X} , \mathcal{U} and $\tilde{\mathcal{Y}}$ are function spaces with $\mathbf{x}(t) \in \mathbb{R}^n$, $y(t) \in \mathbb{R}^p$ and $u(t) \in \mathbb{R}^m$. Note that the function h may or may not be directly dependent on u . If the input u appears in the mapping h explicitly, the model is said to have a “direct feedthrough”.

This work is focused on the single-variable case ($p = m = 1$). The model is assumed to be stable and not to contain a time-delay. The elements of the vector field \mathbf{f} and the function h are analytic functions of their arguments \mathbf{x} and u . The solution $\mathbf{x}(t)$ of (1a) is assumed to exist and to be unique. The behavior of the model $\tilde{\Sigma}$ is described by regarding the entire input, state, and output as signals. In the following, the expressions “signal” and “trajectory” mean a function of time. The transition map $\theta: \mathcal{U} \rightarrow \mathcal{X}$ maps the input space \mathcal{U} into the space \mathcal{X} of trajectories of the states \mathbf{x} , which starts at \mathbf{x}_0 . The initial condition \mathbf{x}_0 is assumed to be given. Thus, the dependence of θ on \mathbf{x}_0 is not shown explicitly, which simplifies the notation. The measurement map $h: \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^p$ maps the current value of the states and input signal into the output signal.

The i/o behavior (i/o map) of the system $\tilde{\Sigma}: \mathcal{U} \rightarrow \tilde{\mathcal{Y}}$ is written by $\tilde{\mathbf{y}} = \tilde{\Sigma}u$, $\forall u \in \mathcal{U}$, where the concatenation operator ‘ \circ ’ is omitted.

The output-function space

$$\tilde{\mathcal{Y}} = \{ \tilde{y} \in \mathcal{Y} : \exists u \in \mathcal{U} : \tilde{y} = \tilde{\Sigma}u \} \quad (2)$$

contains all signals which can be produced by the model $\tilde{\Sigma}$. Thus, the i/o map $\tilde{\Sigma}$ is surjective on this set.

In order to define the system gain, the norms of its i/o signals are introduced. The norm \mathcal{L}_p for a signal u is defined as

$$\|u\|_{\mathcal{L}_p} = \left(\int_t |u|^p dt \right)^{\frac{1}{p}} \quad (3)$$

with $1 \leq p < \infty$. For $p = \infty$ the \mathcal{L}_∞ -norm is

$$\|u\|_{\mathcal{L}_\infty} = \sup_{t \geq 0} |u(t)| < \infty \quad (4)$$

Remark 1 A signal u , for which the norm \mathcal{L}_∞ exists (i.e. is finite), belongs to the space of piecewise continuous bounded functions (cf. [4]).

Definition 1 (Stability, [4]) An i/o map $\tilde{\Sigma}$ is finite-gain \mathcal{L}_p -stable if there exist nonnegative constants γ and β such that

$$\|(\tilde{\Sigma}u)_\tau\|_{\mathcal{L}_p} \leq \gamma \|u_\tau\|_{\mathcal{L}_p} + \beta \quad (5)$$

holds for all $u \in \mathcal{L}_p$ and $\tau \in [0, \infty)$, where u_τ is the truncated signal u :

$$u_\tau = \begin{cases} u(t), & 0 \leq t \leq \tau \\ 0, & t > \tau. \end{cases}$$

Definition 2 (System gain, [4]) For a finite-gain \mathcal{L}_p -stable system, the smallest value γ , for which inequality (5) is satisfied, is called the gain of the system denoted by $g(\tilde{\Sigma}) = \gamma$.

Definition 3 (Steady-state behavior) The term $\tilde{\Sigma}_\infty$ means the i/o behavior of $\tilde{\Sigma}$ at steady-state. It is defined using the steady-state signals $\lim_{t \rightarrow \infty} u(t) = u_\infty < \infty$ and $\tilde{y}_\infty = \tilde{\Sigma}_\infty u_\infty < \infty$ as

$$\tilde{\Sigma}_\infty : \quad \begin{aligned} \mathbf{0} &= \mathbf{f}(x_\infty, u_\infty) \\ \tilde{y}_\infty &= h(x_\infty, u_\infty) \end{aligned} \quad (6)$$

Definition 4 (Relative degree) The relative degree r of a system $\tilde{\Sigma}$ is the smallest value $r \in \mathbb{N}_0^+$ for which $\tilde{y}^{(r)}$ can

be expressed by an algebraic function φ which explicitly depends on the input u :

$$\tilde{y}^{(r)} = L_f^r h(\mathbf{x}) = \varphi(\mathbf{x}, u) \quad (7)$$

with

$$\begin{aligned} \frac{\partial}{\partial u} L_f^i h(\mathbf{x}) &= 0, \quad 0 \leq i \leq r-1 \\ \frac{\partial}{\partial u} L_f^r h(\mathbf{x}) &\neq 0 \end{aligned}$$

where $L_f h(\mathbf{x})$ denotes the Lie derivative of the function h in the direction of the vector field \mathbf{f} :

$$L_f h(\mathbf{x}) = \frac{\partial h(\mathbf{x})}{\partial \mathbf{x}} \mathbf{f}(\mathbf{x}) \quad (8a)$$

$$L_f^k h(\mathbf{x}) = \frac{\partial L_f^{k-1} h(\mathbf{x})}{\partial \mathbf{x}} \mathbf{f}(\mathbf{x}) \quad (8b)$$

Remark 2 The relative degree (sometimes called relative order) can be interpreted as the number of integrations that the input u (or some algebraic function of it) has to undergo until it affects the output \tilde{y} . A relative degree of $r = 0$ implies a system with direct feedthrough, i.e. $\tilde{y} = h(\mathbf{x}, u)$.

Throughout this paper, it is assumed that $r < \infty$ exists and is (at least locally in a neighborhood of x_0) well-defined.

Lemma 1 (Output-function space) Assume a model $\tilde{\Sigma}$ given in (1) with $\mathcal{U} = \mathcal{L}_\infty$ and denote the relative degree by r . Then, the output-function space $\tilde{\mathcal{Y}}$ of the model $\tilde{\Sigma}$ is equal to or a subset of all r -times differentiable functions, i.e.

$$\tilde{\mathcal{Y}} \subseteq C^{r-1} \quad (9)$$

The proof of the Lemma can be found in [3].

Definition 5 (Right inverse [5]) The right inverse $\tilde{\Sigma}^r : \tilde{\mathcal{Y}} \rightarrow \mathcal{U}$ of the system $\tilde{\Sigma}$ is a mapping with the property

$$\tilde{\Sigma} \tilde{\Sigma}^r \tilde{y}_d = \tilde{y}_d = \tilde{y} \quad (10)$$

for all $\tilde{y}_d \in \tilde{\mathcal{Y}}$.

Thus, for every $\tilde{y}_d \in \tilde{\mathcal{Y}}$ the right inverse $\tilde{\Sigma}^r$ generates an input u such that the model output \tilde{y} exactly follows the trajectory \tilde{y}_d , see Figure 1.

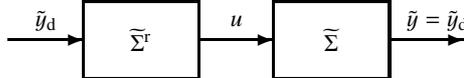


Figure 1
Right inverse.

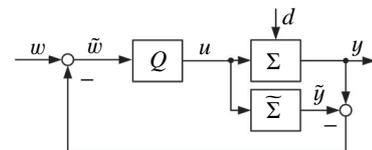


Figure 2
IMC Structure.

1.1.2 IMC Structure

This section gives a survey of the IMC concept and its structural properties. Figure 2 shows the IMC structure with *IMC controller* Q , some nonlinear plant Σ and nonlinear plant model $\tilde{\Sigma}$.

Disturbances d on the plant Σ include additive input and output disturbances as well as internal disturbances which affect the model error.

The control problem considered is to find an IMC controller Q such that the following is achieved robustly:

- The closed-loop is internally stable,
- the plant output y tracks the reference signal $w \in \mathcal{W} = \mathcal{L}_\infty$ with zero steady-state offset ($y_\infty = w_\infty$ for $w_\infty = \lim_{t \rightarrow \infty} w = \text{const}$ and $d_\infty = \lim_{t \rightarrow \infty} d = \text{const}$), where it is assumed that future values $w(\tau)$ with $\tau > t$ are unknown.

1.1.3 Properties of the IMC Structure

This section describes general properties of the IMC structure shown in Figure 2 which apply to any IMC controller Q independently of the design method used. The block diagram in Figure 2 yields

$$\tilde{w} = w - y + \tilde{y} \quad (11)$$

$$\tilde{y} = \tilde{\Sigma} Q \tilde{w} \quad (12)$$

From (11) and (12) the following three properties can be derived [6].

Property 1 (Nominal Stability) Assume a perfect model ($\Sigma = \tilde{\Sigma}$) and the absence of disturbances ($d = 0$). Then, the closed-loop system in Figure 2 is internally stable if the controller Q and the plant $\tilde{\Sigma}$ are stable.

Hence, the stability of the plant is a necessary condition for closed-loop stability.

Property 2 (Perfect Control) Assume that the right inverse of the model $\tilde{\Sigma}^r$ exists and that the closed-loop system is input-output stable with controller $Q = \tilde{\Sigma}^r$. Then, the control will be perfect ($y = w$) for arbitrary disturbances d .

Property 3 (Zero Offset) Assume that the right inverse of the model in steady-state $\tilde{\Sigma}_\infty^r$ exists, that $Q_\infty = \tilde{\Sigma}_\infty^r$ and that the closed-loop system is input-output-stable with such a controller. Then, offset-free control $y_\infty = w_\infty$ is attained for asymptotically constant reference signals $\lim_{t \rightarrow \infty} w = w_\infty$ and constant disturbances d .

Properties 2 and 3 hold despite of model uncertainties. Property 3 implies that if the steady-state gain of the IMC controller is the inverted steady-state gain of the model then there will be no steady-state offset even in the presence of model uncertainties. Thus, it is not necessary (and rather unreasonable) to add an explicit integrator to the IMC controller.

In summary, the IMC structure exhibits some attractive properties like nominal stability and offset-free control. In the following, a design procedure is proposed in which the IMC controller Q is constructed.

1.2 Differential Flatness of Dynamical Systems

1.2.1 Definition of a Flat System

Flatness is a system property. Consider a nonlinear plant model in the state-space representation (1) for the SISO case ($p = m = 1$).

Definition 6 (Flatness [2]) The system $\tilde{\Sigma}$ is called flat if there is a variable $z(t)$ (called the flat output), such that the following conditions are satisfied:

1. The flat output $z(t)$ can be represented in terms of the state $\mathbf{x}(t)$

$$z(t) = \Phi(\mathbf{x}(t)) \quad (13)$$

2. The state $\mathbf{x}(t)$ the input $u(t)$ and their time derivatives can be represented in terms of $z(t)$ and a finite number of its time derivatives $\dot{z}, \dots, z^{(n)}$:

$$\mathbf{x}(t) = \psi_1(z(t), \dot{z}(t), \dots, z^{(n-1)}(t)) \quad (14a)$$

$$u(t) = \psi_2(z(t), \dot{z}(t), \dots, z^{(n)}(t)) \quad (14b)$$

If the conditions (14) are satisfied then the output $y(t)$ can be represented by:

$$y(t) = h(\psi_1(z(t), \dots, z^{(n-1)}(t)), \psi_2(z(t), \dots, z^{(n)}(t))) \quad (15a)$$

If the output map h is not dependent on the input $y = h(\mathbf{x})$, (15a) can in general be expressed by some function h_ψ which may not need derivatives of z up to n .

$$y(t) = h_\psi(z(t), \dots, z^{(q)}(t)), \quad \text{with } q \leq n \quad (15b)$$

The value of $r = n - q$ is called the relative degree which is of importance when designing a flatness-based IMC controller.

The flat output $z(t)$ and its time derivatives $z^{(i)}(t)$ with $i = 1 \dots n$ describe the system dynamics, since their knowledge suffices to compute all the other system variables $\mathbf{x}(t)$, $u(t)$ and $y(t)$.

1.2.2 Feedforward Control Law

The aim of this section is to find a feedforward control law $u(t) = u_d(t)$ for a flat system $\tilde{\Sigma}$, such that the flat output $z(t)$ follows a given trajectory $z_d(t)$ exactly.

Assume that the control requirements are given in terms of the behavior of the flat output $z(t)$ and let this desired behavior be denoted by $z_d(t)$. Further, assume that $z_d(t)$ is $(n - 1)$ -times continuously differentiable and that all $z_d^{(i)}(t)$

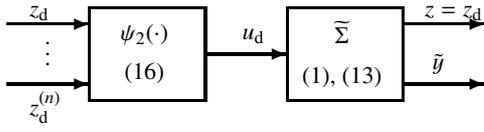


Figure 3
Flatness-based feedforward control structure.

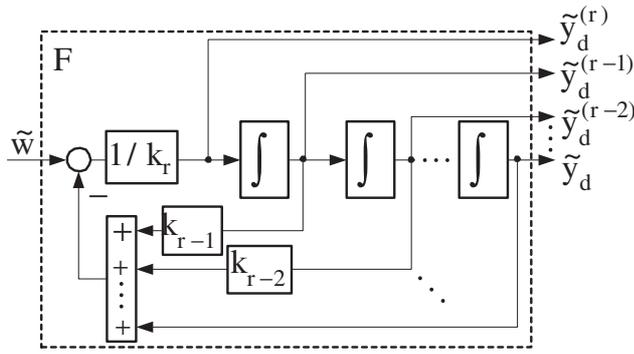


Figure 4
IMC Filter F .

with $i = 0, \dots, n$ are known. Then, the control input $u(t) = u_d(t)$ can be determined by using (14b):

$$u_d(t) = \psi_2(z_d(t), \dots, z_d^{(n)}(t)) \quad (16)$$

Here, (16) is called the *flatness-based feedforward control law*.

The main result of this section is a perfect feedforward controller:

Proposition 1 (Perfect Feedforward Controller) *A given n -times differentiable trajectory $z_d(t)$ with known derivatives $z_d^{(i)}(t)$ (with $i = 1, \dots, n$) is achieved by the system $\tilde{\Sigma}$ exactly*

$$z(t) = z_d(t) \quad (17)$$

by the control input $u(t) = u_d(t)$ from (16) if the initial condition of the given trajectory $z_d(t)$ matches the initial condition of the system.

Figure 3 shows the resulting flatness-based feedforward control structure.

Corollary 1 suggests that (16) is the right inverse [3] of $\tilde{\Sigma}$ with respect to its flat output z for given n -times differentiable trajectories.

1.3 Flatness-Based IMC

This section shows how the two approaches from the previous section, namely the IMC structure and a flatness-based feedforward control, can be combined.

1.3.1 Trajectory Generation

The feedforward law (16) requires the values of $z_d(t)$ and its n derivatives. In order to use the flatness-based feedforward control as feedback Q in the IMC loop, these values have to be determined from the current $w(t)$ and its past values. Furthermore the reference trajectory w may be any arbitrary bounded signal, including step functions. According to Lemma 1, the set $\tilde{\mathcal{Y}}$ of possible output signals of the model $\tilde{\Sigma}$ is a part of the set of $r - 1$ -times differentiable functions. Consequently, in general $w \notin \tilde{\mathcal{Y}}$ must be assumed and some way is required to map the signal w to a trajectory that is realizable by the plant. Both problems are addressed by the following steps.

IMC Filter

The IMC filter F filters the reference signal $\tilde{w}(t)$ such that the filter output $\tilde{y}_d(t)$ can be achieved by the model output $\tilde{y}(t)$. Additionally, this filter will give the first r derivatives of $\tilde{y}_d(t)$, namely $\tilde{y}_d^{(i)}(t)$ for $i = 0, \dots, r$. For F a linear filter is proposed here. If the plant model $\tilde{\Sigma}$ is linear, this approach is equivalent to the classical IMC controller design [7]. Any filter algorithm is feasible including nonlinear filters as long as it determines r derivatives $\tilde{y}_d^{(i)}(t)$ of its input signal $\tilde{w}(t)$. The types of trajectories used and their generation differs depending on the control problem (cf. [8]).

Suppose, the filter F is designed as a linear filter with the following transfer function

$$F(s) = \frac{1}{k_r s^r + k_{r-1} s^{r-1} + \dots + 1} \quad (18)$$

then, it can be implemented as a state-variable filter as shown in Figure 4. The filter F is similar to the filter used in linear IMC controllers. It only differs from the classical IMC filter in such that it also gives r derivatives of the output.

Mapping y to z

The filter F delivers the demand $\tilde{y}_d(t) \in \tilde{\mathcal{Y}}$ on the system output $y(t)$, together with its r derivatives. These can be mapped to the demand $z_d^{(j)}(t)$ with $j = 0, \dots, n$ on the system's flat output $z(t)$, which is used by the flatness-based feedforward control law (16). This procedure will be denoted by $F_{y \rightarrow z}$.

The relationship between $\tilde{y}_d(t)$ and $z_d(t)$ is obtained by using (15). The case $q < n$ is discussed in the following:

$$\tilde{y}_d(t) = h_\psi(z_d(t), \dots, z_d^{(q)}(t)), \quad \text{with } q < n \quad (19a)$$

Equation (19a) presents a dynamic relationship between $\tilde{y}_d(t)$ and the desired flat output $z_d(t)$. It is a differential equation which needs to be solved for $z_d(t)$. For simplicity it is assumed that the solution is stable and can be obtained numerically.

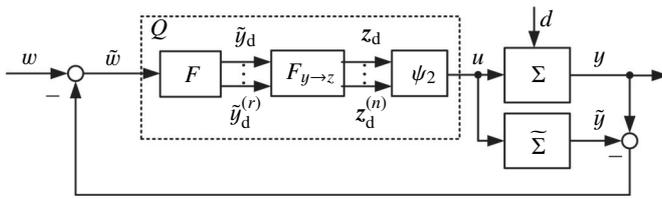


Figure 5

Flatness based IMC structure.

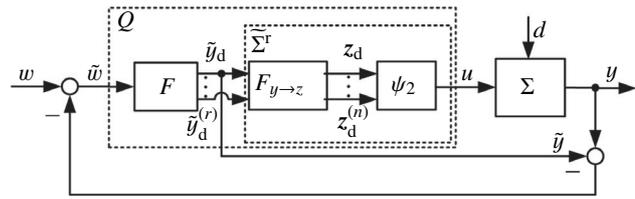


Figure 6

Flatness based IMC controller without internal model.

However, the control law (16) requires all derivatives $z_d^{(j)}(t)$ with $j = 0, \dots, n$ but a numerical solution of (19a) would only yield $z_d^{(q)}(t)$ as highest derivative. To this end, additional r derivatives of (19a) are calculated:

$$\dot{\tilde{y}}_d(t) = \frac{d}{dt} \left(h_\psi \left(z_d(t), \dots, z_d^{(q)}(t) \right) \right) \quad (19b)$$

$$\vdots$$

$$\tilde{y}_d^{(r)}(t) = \frac{d^r}{dt^r} \left(h_\psi \left(z_d(t), \dots, z_d^{(q)}(t) \right) \right) \quad (19c)$$

The solution of the differential equation (19c) will yield the derivatives of the flat output up to $z_d^{(n)}(t)$. The possibly remaining lower derivatives $z_d^{(l)}(t)$ with $l = 0, \dots, r-1$ can be obtained either by integration or by the relationships (19).

Remark In the following, mapping a demand on y to z by the process of (numerically) solving (19c) and then obtaining the necessary $z_d^{(j)}(t)$ with $j = 0, \dots, n$ is denoted by $F_{y \rightarrow z}$.

1.3.2 IMC Controller

The results of the sections above can be used to obtain a feedforward controller Q for the plant model $\tilde{\Sigma}$: the flatness-based feedforward control law ψ_2 from (16) requires the signals $z_d^{(j)}(t)$ with $j = 0, \dots, n$ as inputs, which are obtained from $F_{y \rightarrow z}$. The sequence of $\psi_2 F_{y \rightarrow z}$ is the right inverse of $\tilde{\Sigma}$ with respect to its output \tilde{y} . However, this sequence is not realizable since $F_{y \rightarrow z}$ requires r derivatives of the reference signal $w(t)$. They are supplied by the IMC filter F . The structure of the resulting feedforward controller Q is shown in Figure 5, where it has already been integrated into the IMC structure.

When the IMC controller Q is used in the IMC structure, the input signal is changed from w to

$$\tilde{w} = w - (y - \tilde{y}) \quad (20)$$

The resulting flatness-based IMC structure in Figure 5 is a nonlinear control loop which is *not* based on linearization. The behavior of $\tilde{\Sigma}Q$ (controller to model) is identical to F . In the case of a multivariable system, this procedure will not decouple the inputs. The IMC structure does not change the

nonlinear character of the system but still achieves robust tracking performance and stability.

The flatness-based IMC controller in Figure 5 is a one-degree-of-freedom controller with the freedom to change the parameters of the filter F in (18). In the face of model uncertainties, the gain of Q needs to be small enough to guarantee overall stability (see [3] for details). Therefore, the filter F needs to be slow enough for the closed-loop to be stable and fast enough to achieve satisfying performance.

The design idea can be summarized as starting with a fast feedforward controller and then decreasing its speed to guarantee stability in the presence of model uncertainties and disturbances. This design results in a trade-off between robustness and performance. However, this paper focuses on the general idea of this feedback controller and will not discuss the design procedure any further.

1.4 Substitution of the Internal Model

In Figure 5 the plant model $\tilde{\Sigma}$ is simulated online in order to generate the model output $\tilde{y}(t)$. However, since the desired behavior of the model is defined through

$$\tilde{y} = \tilde{y}_d = F \tilde{w} \quad (21)$$

and the input u is computed according to (16) such that the desired behavior is achieved *exactly*, the model can be substituted as shown in Figure 6.

Both structures in Figure 5 and Figure 6 have the same behavior, since both generate the same $\tilde{y}(t)$. With the substitution of the model the overall complexity of the control structure is reduced significantly.

1.5 Calibration of an IMC

In order to perform a calibration of a flatness-based IMC controller, it is assumed, that a flatness-based feedforward controller was computed and implemented in the IMC control structure as displayed in Figure 5 or Figure 6.

The calibration procedure can be split into two parts:

- Adjusting the degrees of freedom, namely the poles of the IMC filter F .

- Adjusting the model $\tilde{\Sigma}$. Although most model parameters can be found off-line by parameter estimation procedures using measurement data, some parameters, like I/O characteristics are best determined online at the testbed. However, if necessary, all parameters can be identified at the testbed and they can be changed at any time in the future.

If the filter F is implemented as

$$F = \frac{1}{(s/\lambda + 1)^r} \quad (22)$$

only one parameter, namely λ , defines the compromise between performance and robustness. From the structural IMC properties it is clear that, if the model was perfect, the filter F could be arbitrarily fast without sacrificing stability. In [3] it is outlined how an upper limit on the speed of F can be estimated for the case of an imperfect model and uncertainties.

The model $\tilde{\Sigma}$ in the IMC control loop represents the plant. If the model is derived using physical relationships it will consist of a number of physical parameters (like mass, stiffness, efficiency, etc.). If the structure of the model includes all major effects, then there must be a choice of parameters such that the model represents the plant very well. These parameters can be determined on a testbed by experts on the plant by a variety of means. For this, no control engineer is necessary.

The IMC controller (which is now the flatness-based feedforward controller) also contains model parameters, which are changed automatically during calibration. If the model represents the plant very well, the closed-loop system will behave very closely to the filter F .

The combination of good model parameters and the choice of a single control parameter, namely λ , determines the closed-loop performance and robustness. Therefore, calibration can be performed by non control engineers and at any point in the design process. Calibration of a flatness-based IMC focuses on finding model parameters and thereby respects the separation of control algorithm design and calibration. Additionally, flatness-based IMC incorporates robust properties like stability and zero steady-state offset.

In the following section, an important extension of the flatness-based IMC structure is presented, which addresses the very common problem of a plant with input limitations.

2 IMC FOR A SYSTEM WITH INPUT CONSTRAINTS

In all existing plants (e.g. machinery, chemical processes) the accessible inputs $u \in \mathcal{U}$ are bounded. This section introduces the main theoretical result of this paper, namely an extension of the flatness-based IMC structure to respect such input constraints. For most common control design principles explicit consideration of input saturations in the design process is not possible. Thus, they have to rely on

anti-windup or similar techniques to avoid instability. The IMC structure in contrary allows for a very elegant solution to the problem by considering the input constraints in the trajectory planning, resulting in a control structure that exploits the maximum dynamic range of a system in terms of admissible inputs, without ever violating input bounds.

The following introduces a procedure that allows mapping of the plant's input constraints to a limitation of the r -th derivative of the model output \tilde{y} . A modification of the IMC filter F is proposed that limits the derivative $\tilde{y}_d^{(r)}$ of the generated trajectory such that it is achievable within permissible inputs.

2.1 Mapping Input Constraints to the Output

Assume that the admissible inputs of the plant Σ are bounded by

$$\mathcal{U} = \{u \mid u_{\min} \leq u \leq u_{\max}\} \quad (23)$$

For a model with relative degree r , the input u directly affects the derivative $\tilde{y}^{(r)}$ of the model output [3]. Thus, at any time t there exist a maximal and minimal value for the highest derivative $\tilde{y}^{(r)}(t)$ that are possible for the model output to obtain within the input range (23). If these bounds for $\tilde{y}^{(r)}(t)$ are known, they can be used to limit the highest derivative of \tilde{y}_d in the IMC filter F which in turn will plan the desired trajectory such that it is achievable within permissible inputs.

For flat systems, the bounds for $\tilde{y}^{(r)}(t)$ can be obtained by first finding maximum and minimum values for the derivative $z_d^{(n)}$ of the flat output. The feedforward control law (16) can be solved for $z_d^{(n)}$ which results in some function ξ such that

$$z_d^{(n)} = \xi(z_d, \dot{z}_d, \dots, z_d^{(n-1)}, u) \quad (24)$$

holds. The bounds on the highest derivative of the flat output can be found by

$$z_{d,\max}^{(n)}(t) = \max_{u \in \mathcal{U}} \left\{ \xi(z_d(t), \dots, z_d^{(n-1)}(t), u) \right\} \quad (25a)$$

$$z_{d,\min}^{(n)}(t) = \min_{u \in \mathcal{U}} \left\{ \xi(z_d(t), \dots, z_d^{(n-1)}(t), u) \right\} \quad (25b)$$

These constraints on $z_d^{(n)}$ can be mapped to constraints for the highest derivative of the IMC filter output $\tilde{y}^{(r)}$. Equation (19c) gives a relationship for $\tilde{y}_d^{(r)}$, which can be written as some function φ of z_d and its n derivatives:

$$\tilde{y}_d^{(r)} = \frac{d^r}{dt^r} (h_\psi(z_d, \dots, z_d^{(q)})) =: \varphi(z_d, \dots, z_d^{(n)}) \quad (26)$$

The function φ can be used to map the limits (25) to

$$\tilde{y}_{d,\lim1}^{(r)}(t) = \varphi(z_d(t), \dots, z_d^{(n-1)}(t), z_{d,\max}^{(n)}(t)) \quad (27a)$$

$$\tilde{y}_{d,\lim2}^{(r)}(t) = \varphi(z_d(t), \dots, z_d^{(n-1)}(t), z_{d,\min}^{(n)}(t)) \quad (27b)$$

from which the actual upper and lower bound can be determined by

$$\tilde{y}_{d,\max}^{(r)}(t) = \max\{\tilde{y}_{d,\text{lim}1}^{(r)}(t), \tilde{y}_{d,\text{lim}2}^{(r)}(t)\} \quad (28a)$$

$$\tilde{y}_{d,\min}^{(r)}(t) = \min\{\tilde{y}_{d,\text{lim}1}^{(r)}(t), \tilde{y}_{d,\text{lim}2}^{(r)}(t)\} \quad (28b)$$

Independently of the type of IMC filter that is used, if it is designed such that the constraints (28) are respected for the highest derivative, then the feedforward law (16) will compute a control input that respects the input constraints *exactly*. It is important to note that this also holds in the face of modeling errors, as long as the feedforward law and the bounds (28) are determined for the same model $\tilde{\Sigma}$.

2.2 Extended IMC Filter for Input Constraints

The resulting bounds $\tilde{y}_{d,\min}^{(r)}(t)$, $\tilde{y}_{d,\max}^{(r)}(t)$ are dynamic, since they depend on the actual values of $z_d(t)$ and its $n-1$ derivatives, which can be obtained on-line from $F_{y \rightarrow z}$. Thus, they can be computed in realtime and used to limit the highest derivative in the IMC filter F such that

$$\tilde{y}_{d,\min}^{(r)}(t) \leq \tilde{y}_d^{(r)}(t) \leq \tilde{y}_{d,\max}^{(r)}(t) \quad (29)$$

It is proposed to extend the state-variable filter from Figure 4 by a nonlinear dynamic saturation. If this filter is used in the flatness-based IMC structure from Figure 5 or Figure 6, then input constraints will be respected by the controller at all times and no anti-windup is required.

3 BOOST PRESSURE CONTROL FOR A TURBOCHARGED DIESEL ENGINE

The example of boost pressure control of a turbocharged engine is used to demonstrate the capabilities of the flatness-based IMC approach, including the consideration of input constraints.

3.1 Control Problem

Figure 7 shows an airpath through a turbocharged engine. Ambient air enters the system through the air box (1) and is compressed by the compressor (2). Much of the gained temperature is reduced by the charge air cooler (4) to increase the air density. The compressed air enters the engine (5) and is mixed with fuel. The mixture is then combusted creating torque. The resulting exhaust stores a lot of energy in terms of heat (enthalpy) and pressure. This energy is used by the turbine (6) to generate mechanical work which drives the compressor (2) via the shaft (3). The assembly of (2), (3), and (6) is called a turbocharger. Here, the turbine (6) is a variable-nozzle turbine (VNT). The nozzles are adjusted by a vacuum-controlled diaphragm box (7). The exhaust then flows through the exhaust pipe through catalytic converters and particulate filters (exhaust after-treatment (8)).

Engine speed n and injected fuel mass per second \dot{m}_f have a major influence on the system and can be considered as measured disturbances $\mathbf{d} = [n, \dot{m}_f]^T$, which depend on road conditions and the driver.

The control goal is to set the nozzle position (input u) such that the boost pressure (output y) follows its reference signal w as closely as possible.

3.2 Model of the Plant

Many physically motivated models of a turbocharged diesel engine have been introduced in the literature (*e.g.* [9]). For control applications the engine can be represented by a mean-value model, and all pipes and chambers in the airpath can be modeled by the filling- and emptying method. The model used here originates from the approach [10].

However, some simplification concerning the thermodynamics were made. It is assumed that the gas properties do not change with composition and temperature, that pressure and temperature are uniform over a plenum chamber and, since the flow velocities are small, the difference between static and total temperature and pressure is ignored.

Here, this simplified model is called the design model. It is used as IMC model $\tilde{\Sigma}$ and as the basis for developing a flatness-based feedforward controller. The design model is described by using the state vector $[\omega, p]^T$ and the input u , where ω is the speed of the turbocharger and p is the boost pressure. The input u is a function of the nozzle position of the variable nozzle turbine (VNT) and bounded by

$$u_{\min} \leq u(t) \leq u_{\max} \quad \forall t \quad (30)$$

where u_{\min} and u_{\max} depend on the maximum and minimum nozzle positions. The design model $\tilde{\Sigma}$ is given by

$$\tilde{\Sigma} : \dot{\omega} = \left(\frac{k_{32}k_{33}}{\omega} (k_{43}(n, \dot{m}_f) + k_{42}(n)k_{10}p) \varphi_1(p) \right) (1-u) - \frac{k_{32}k_{24}K_I(p)}{\omega} \varphi_2(\omega, p) \quad (31a)$$

$$\dot{p} = \frac{K_{II}(p)}{k_{72}} \varphi_2(\omega, p) - \frac{k_{73}}{k_{72}} k_{42} p \quad (31b)$$

and the output equation

$$y = p \quad (31c)$$

with

$$\varphi_1(p) = \frac{k_{73}}{k_{10}} + \frac{k_{45}(n, \dot{m}_f)}{k_{47}(n, \dot{m}_f) + k_{10}p} \quad (31d)$$

$$\varphi_2(\omega, p) = k_{21} \frac{k_{22}\omega^2 - K_I(p)}{k_{23}\omega} \quad (31e)$$

$$K_I(p) = k_{25} \left(\left(\frac{p}{p_{\text{amb}}} \right)^{k_{39}} - 1 \right) \quad (31f)$$

$$K_{II}(p) = k_{27}K_I(p) + k_{28} \quad (31g)$$

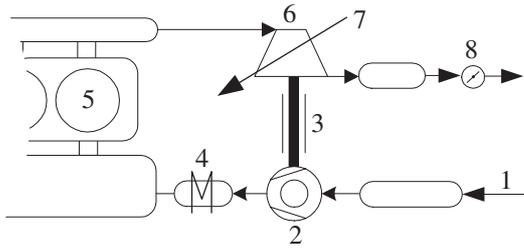


Figure 7
Air path of a turbocharged diesel engine.

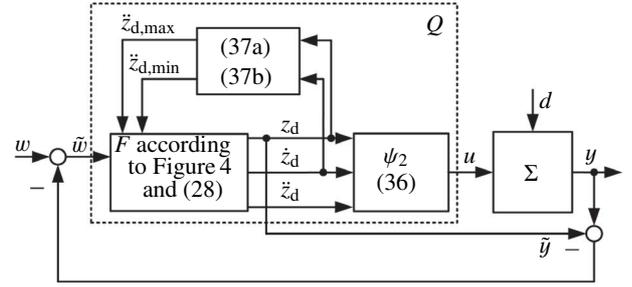


Figure 8
Complete IMC structure.

where the coefficients k_{ij} are system parameters (e.g. diameters, inertia of the turbocharger, etc.), p_{amb} is the ambient pressure and the $\varphi_j(\cdot)$ are some nonlinear functions of the states.

The nonlinear model (31) is a flat system with $z = y = p$ as its flat and measured output, which is shown in the following.

3.3 Flat Output

For the design model (31) it is shown that the boost pressure p is a flat output (which also happens to be the system output y)

$$z = y = p \quad (32)$$

In order to show the flatness of the system, all state variables as well as the input have to be expressed as a function of z and a finite number of its time derivatives. By differentiating (32), one gets

$$\begin{aligned} \dot{z} &= \frac{K_{II}(p)}{k_{72}} \varphi_2(\omega, p) - \frac{k_{73}}{k_{72}} k_{42}(n) p \\ &= \frac{K_{II}(z) k_{21} (k_{22} \omega^2 - K_I(z))}{k_{72} k_{23} \omega} - \frac{k_{73}}{k_{72}} k_{42}(n) z \end{aligned} \quad (33)$$

The quadratic equation (33) for ω has the following positive unique solution for $z > p_{amb}$, i.e. $p > p_{amb}$:

$$\begin{aligned} \omega &= \frac{k_{72} k_{23} \dot{z} + \frac{k_{73}}{k_{72}} k_{42}(n) z}{2 k_{21} k_{22} K_{II}(z)} \\ &+ \frac{\sqrt{(k_{72} k_{23} \dot{z} + \frac{k_{73}}{k_{72}} k_{42}(n) z)^2 + 4 k_{21}^2 k_{22} K_I(z) K_{II}(z)}}{2 k_{21} k_{22} K_{II}(z)} \end{aligned} \quad (34)$$

Using the second derivative of (32) and taking (34) into account, one gets

$$\ddot{z} = \varphi_4(z, \dot{z}, n, \dot{m}_f) + \varphi_5(z, \dot{z}, n, \dot{m}_f) u \quad (35)$$

which leads to the feedforward control law:

$$u = \frac{\ddot{z} - \varphi_4(z, \dot{z}, n, \dot{m}_f)}{\underbrace{\varphi_5(z, \dot{z}, n, \dot{m}_f)}_{\psi_2(z, \dot{z}, n, \dot{m}_f)}}; \quad \varphi_5(z, \dot{z}, n, \dot{m}_f) \neq 0 \quad (36)$$

A solution exists for all z, \dot{z} , if $z > p_{amb}$. The feedforward control law is described by (36) with $z = z_d$ from which the open and closed-loop control are derived in the following.

3.4 Trajectory Planning

Since measured output y and flat output z are identical, trajectory planning can be done directly from w to the flat output z_d (i.e. no differential equation like (15b) has to be solved). Trajectory planning needs to create the trajectory for the flat output $z_d(t)$ and its first two derivatives $\dot{z}_d(t)$ and $\ddot{z}_d(t)$ from the reference signal $w(t)$. To that end, a second order filter F is used. Because the system is subject to input constraints, the highest derivative of the filter has to be saturated. The necessary computation are addressed in the next subsection.

3.5 Input Constraints

The trajectory for $z_d(t)$ needs to be planned such that the input constraints (30) are respected. The bounds on $u(t)$ can be mapped to bounds on the highest derivative of the flat output, $\ddot{z}_d(t)$, using (35):

$$\ddot{z}_{d,max} = \varphi_4(z_d, \dot{z}_d, n, \dot{m}_f) + \varphi_5(z_d, \dot{z}_d, n, \dot{m}_f) u_{max} \quad (37a)$$

$$\ddot{z}_{d,min} = \varphi_4(z_d, \dot{z}_d, n, \dot{m}_f) + \varphi_5(z_d, \dot{z}_d, n, \dot{m}_f) u_{min} \quad (37b)$$

The above relations always hold, because $\varphi_5(z, \dot{z}, n, \dot{m}_f) > 0 \forall t$. Because in this case the flat and the actual output are identical, no further mapping of the bounds is needed. They can be employed directly to limit the highest derivative in the IMC filter F , which needs to be extended by a dynamic saturation according to Figure 4 taking (28) into account. It is important to note that (37) constitute dynamic bounds, due

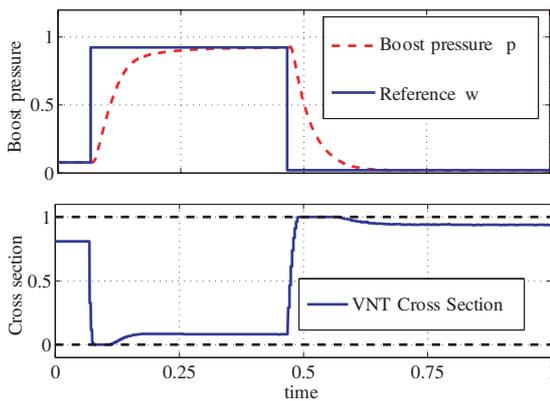


Figure 9

Simulation results of the boost pressure control of a full-scale air system model.

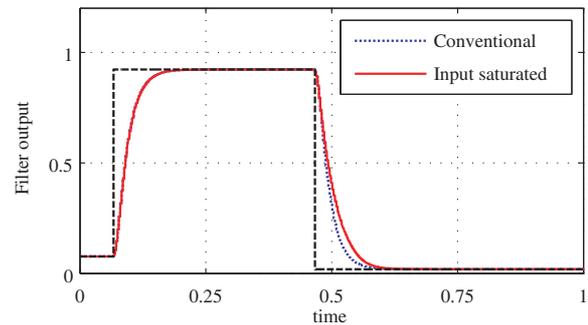


Figure 10

Filter output with and without input saturation in the nominal case, *i.e.* controlling the design model (31).

to their dependence on the actual values of z_d , \dot{z}_d , n , \dot{m}_f and thus need to be computed on-line and provided to the IMC filter. While the external signals n , \dot{m}_f are measured, the values of z_d and \dot{z}_d can be obtained directly from the filter output.

Figure 8 shows the complete IMC structure for the engine air system Σ with IMC Filter F, feedforward law ψ_2 , and the computation of the boundaries (37). Note that the measured disturbances n , \dot{m}_f are also required by the feedforward law and the boundary computation, although their signal paths have been omitted in the figure.

4 SIMULATION RESULTS

The flatness-based IMC structure from the previous sections is evaluated in simulations. The controlled plant is a full-scale model of a turbocharged engine air system, which is known to represent the behavior of a real engine very well. The plant model has nine states, thus, significant modeling errors with respect to the simplified design model (31) are present. Figure 9 shows the response of the controlled boost pressure p to a step increase and a step decrease of the reference value w , as well as the actual control input of the plant, which is the effective cross-section of the variable nozzle turbine. All values are normalized for reasons of confidentiality. The engine is driven at upper load by a given speed and fuel mass. The simulation results demonstrate the impressive performance of the flatness-based IMC structure. It can be observed in Figure 9 that the trajectory is planned such that the input constraints are exactly respected. Thus, the system is driven at its maximum possible speed during the transients.

For a better understanding of the effects of the saturated IMC filter for the planned trajectory, the nominal case is considered, *i.e.* controlling the design model (31). For this Figure 10 shows the saturated (*cf.* Fig. 4 taking (28) into account) and the conventional (*cf.* Fig. 4) filter response to the reference signal for the nominal case. Note that these responses are *not* identical to the closed-loop filter output in the simulation run from Figure 9, since there the filter output also compensates for modeling errors. Figure 10 shows the signals the filter would deliver if the model is perfect. The response of a conventional PT_2 filter is compared to the output of a filter with identical time constant, but with the saturation structure. For the step increase, the signals are almost identical. Actually the transient of the saturated filter is slightly slower, however this effect is too marginal to be noticed in the plot. For the step decrease however, a significant re-planning of the trajectory occurs at the time where the maximum cross section of the VNT is too small to reach the reference value fast enough, *i.e.* the system acts on its physical limits.

5 TESTBED RESULTS

The results of an engine test stand are presented to portray the capabilities of this concept much better than only simulation results. All unmeasured disturbances act upon the plant and the degree of freedom (the pole λ) was calibrated online on the engine test stand to set it for the actual disturbances. In this case calibration of λ was performed manually. Most model parameters were identified off-line using manufacturer data and additional measurements. Only the characteristic of the vacuum-controlled diaphragm box acting as the VNT-actuator was adjusted at the testbed.

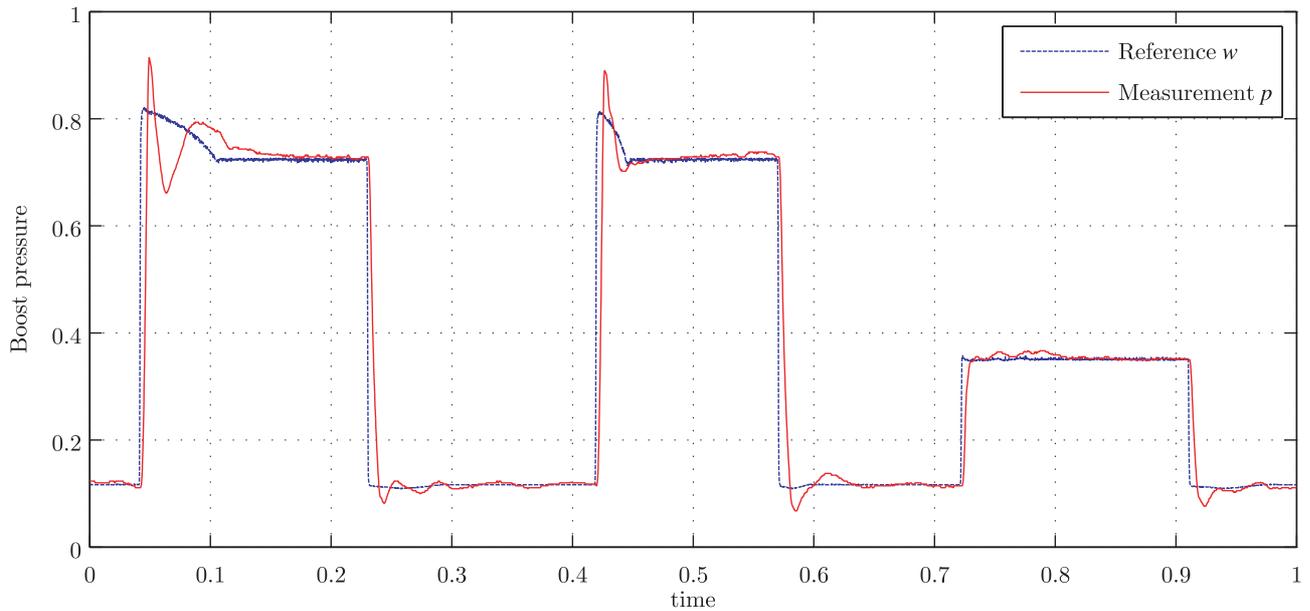


Figure 11

Measurement results of the nonlinear IMC controller for the boost pressure p .

In Figure 11 measurement results on the engine test stand are shown using the flatness-based IMC controller as introduced above. The figures portray the closed-loop performance after calibration. The results are normalized in both axes for reasons of confidentiality. The test run is chosen such that the engine runs in different operation modes with the introduced IMC controller.

The results are very promising since despite the nonlinearity of the plant and controller the basic properties of the IMC structure are retained. Despite some modeling error due to the simplified mean-value filling-and-emptying method, the measured disturbances n and \dot{m}_f and unmeasured disturbances as measurement noise, there never is a steady-state offset. Performance of the closed-loop is at least comparable to the common calibrated gain-scheduled *PID* control structure. However, calibration of a flatness-based IMC controller is significantly more efficient.

CONCLUSION

A new control concept was introduced which is the combination of the IMC control structure with a flatness-based feedforward controller. The introduced concept allows, in an easy way, to take input constraints into account. This control concept fulfills all major requirements of the automotive industry in addition to the usual feedback properties of robust stability and steady-state offset. Compared to today's predominant controllers calibration effort for a boost pressure controller was reduced while at least maintaining

performance. This advantage is gained at the expense of an intense modeling and controller development phase, but the overall design effort has been reduced. Simulation and experimental results for the tracking problem of the boost pressure of a diesel engine are presented. The results show good tracking performance for this nonlinear control problem, while even considering constraints of the control inputs explicitly.

Since this method is still at research level, future developments are expected to deliver superior control performance. A two degree of freedom flatness-based IMC controller has already been developed and shows excellent results in simulations. The IMC feedback method is also valid for non-flat nonlinear systems. It can be shown that all stable nonlinear systems which are input-affine can be controlled using IMC [3].

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Final manuscript received in October 2006

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