Oil & Gas Science and Technology – Rev. IFP, Vol. 62 (2007), No. 2, pp. 137-146

DOI: 10.2516/ogst:2007012

IFP International Conference
Rencontres Scientifiques de l'IFP

Quantitative Methods in Reservoir Characterization
Méthodes quantitatives de caractérisation des réservoirs

Upscaling Fractured Media and Streamline HT-Splitting in Compositional Reservoir Simulation

S. Oladyshkin, S. Skachkov, I. Panfilova and M. Panfilov

Institut National Polytechnique de Lorraine,
Laboratoire d'Energétique et de Mécanique Théorique et Appliquée - LEMTA UMR 7563
Ecole Nationale Supérieure de Géologie, 54501 Vandoeuvre-les-Nancy - France
e-mail: sergey@rdr.leeds.as.uk - irina.panfilova@ensg.inpl-nancy.fr - mikhail.panfilov@ensg.inpl-nancy.fr

Résumé — Changement d'échelle en réservoirs fracturés et HT-décomposition selon les lignes de courant pour un écoulement compositionnel — Nous présentons deux approches destinées à accélérer les simulations de réservoirs. La première approche concerne le calcul de la perméabilité équivalente des milieux fracturés. Le problème basique d'un changement d'échelle qui concerne la formulation des conditions aux limites pour le probléme cellulaire, est résolu à l'aide d'une décomposition du champ global de pression en deux composantes, dont une peut être négligée, tandis que la deuxième est calculée analytiquement. De plus, nous avons développé une méthode rapide de résolution du problème cellulaire qui est basée sur la séparation de la contribution des fractures et de la matrice. Cette méthode amène à des solutions analytiques.

La deuxième approche est consacrée à des simulations *streamline* pour un écoulement compositionnel. L'efficacité de toute simulation *streamline* dans le cas compositionnel est réduite à cause de l'absence de solutions rapides analytiques des problèmes 1D d'écoulement dans ce cas. Nous avons développé un nouveau modèle asymptotique compositionnel qui assure la séparation totale de la thermodynamique et de l'hydrodynamique et aboutit à des solutions analytiques ou mi-analytiques selon les lignes de courant.

***Abstract — Upscaling Fractured Media and Streamline HT-Splitting in Compositional Reservoir Simulation** — We present two approaches devoted to speeding up reservoir simulations. The first approach deals with permeability upscaling for fractured media. The basic problem of any upscaling procedure, which consists of how to formulate the boundary-value conditions for a cell problem, is solved by the method of splitting the global pressure field into two components in such a way that one component may be neglected, while the second one may be calculated in the analytical way. In addition to this, we have developed a new fast method of solution to the cell problem, which is based on splitting the contribution of fractures and a tight matrix.*

The second approach is devoted to streamline simulations for a compositional flow. The low efficiency of any streamline simulation in this case is reduced due to the lack of fast analytical solutions to 1D flow problems. We suggest a new asymptotic compositional flow model which ensures a total splitting between the thermo- and hydrodynamics. As a result, such a splitting leads to an analytical or semi-analytical solution for the multicomponent flow problem along the streamlines.

INTRODUCTION: UPSCALING THE IRREGULAR FIELDS IN TRUE RESERVOIRS

In the present paper we discuss two approaches leading to speeding up numerical reservoir simulations. The first approach concerns the methods of upscaling the permeability field, while the second approach deals with the streamline simulation technique. These two approaches are presented in two sections of the present paper.

About the Problem of Upscaling

In the first section, we examine the classical problem of the upscaling theory. A geological model of a heterogeneous porous reservoir consists of fine-scale cells forming a discrete permeability field. This reservoir is then covered by a numerical hydrodynamic grid of a scale much larger than the geological grid, but much smaller than the overall reservoir size. Approximately, a hydrodynamic cell is of the order of a semi-distance between two wells or even smaller. Such a scale will be called the mesoscale. We need to average the permeability field within each hydrodynamic cell, which constitutes the classic problem of upscaling. Therefore, the objective is to transform a permeability field defined on a fine-grid scale into a coarser grid imposed by a numerical flow simulator.

A problem of upscaling, being based on averaging procedures, is related homogenization theory [1-3]. At the same time, upscaling is different from homogenization. Homogenization provides the averaged permeability of a representative elementary volume (REV), which is an intrinsic medium fragment, while upscaling has the objective the averaged permeability of a medium fragment (a hydrodynamic cell) imposed by exterior factors. The result of homogenization is called "effective permeability'', while upscaling yields "equivalent permeability''. In contrast to effective permeability, equivalent permeability is not an intrinsic medium property and depends on the hydrodynamic grid size and form, as well as on the boundary conditions of the cell.

All the methods of calculating equivalent permeability consist of using the so-called cell problem resulting from the homogenization theory, but imposing different boundary conditions than those used for an effective permeability. A classic cell problem for effective permeability is formulated as a PDE with periodic boundary conditions for some auxiliary functions. For equivalent permeability, the boundary conditions for the cell problem are a priori unknown as they depend on the overall pressure field. Due to this, the equivalent permeability is not strictly defined. It is clear that this value should be determined in such a way that the consecutive hydrodynamic simulations would be equivalent to the reality, which is, however, not a constructive mathematical definition. Due to this uncertainty in the boundary-value conditions, all the methods of the equivalent permeability calculation should be considered as approximated with an uncertain degree of the error committed.

In the first part of Section 1 we present the method of the analytical boundary. It permits one to solve analytically the single-phase problem for the global pressure in a homogeneous medium by the complex potential method. These data are used to formulate the boundary conditions for each cell problem. The obtained cell problems with non-periodic boundary conditions are then analyzed in the second part of Section 1 in order to develop a fast method of solution for a fractured medium. We have developed a new fast analytical method based on the fundamental property of double porosity media.

About the Streamline Technique and Compositional Flow

In Section 2 we analyze a streamline method able to simulate the compositional flow.

The basic advantage of a streamline simulation consists of using analytical 1D solutions to the flow problems along the streamlines. Unfortunately, for a compositional flow the analytical solutions do not exist even in a 1D version. We note that a compositional model consists of N equations of mass balance for each component, two equations of the phase state and $N + 2$ equations of phase equilibria relating the chemical potentials of each component in both phases, phase pressures and phase temperatures. Usually such a model represents a high-order non-linear transcendent equation system which may only be solved numerically using various iteration procedures. In the scope of these iteration procedures, the thermodynamic non-linear high-order subsystem, which should be solved at each space point and in each time instant, frequently consumes a major part of the CPU. Unfortunately, this highly cumbrous part remains invariable whatever the space dimension for the hydrodynamic equations. These circumstances highly complicate the problem solution and do not ensure the efficiency of a compositional streamline simulator.

We propose an effective solution to this problem by splitting the thermodynamic and the hydrodynamic parts along the streamlines. This result is obtained due to some asymptotic properties of the gas-liquid flow, in particular to a high difference between phase mobilities. Due to such a splitting, we obtain an effective semi-analytical solution to the two-phase compositional problem along streamlines. To obtain the analytical solutions for the hydrodynamic part, we have developed a special singular perturbation method based on matching the asymptotic expansions obtained for different space subdomains. To calculate compositional flow along each streamline it is then sufficient to simulate the thermodynamic part once and to insert the obtained thermodynamic functions into the analytical hydrodynamic solution.

1 SEMI-ANALYTICAL UPSCALING FOR A HETEROGENEOUS PERMEABILITY FIELD

We suggest two approaches which are devoted to upscaling the permeability of a fractured medium: the first one (the analytical boundary method) concerns the global improvement in the definition of the boundary-value conditions to the cell problem which defines the equivalent permeability. The second one (splitting the matrix-fracture contribution) concerns the effective solution to this cell problem. Both these questions are the key points of the upscaling procedure.

1.1 Method of the Analytical Boundary

As mentioned, the definition of the equivalent permeability may be done in terms of a boundary-value problem for an intermediate function (“the cell function''). This function is almost equivalent to that which results from the homogenization theory, but with undefined boundary-value conditions for the cell problem. The definition of the most boundary-value conditions for the cell problem is the first point solved in the present section.

The basic principle for imposing the boundary-value conditions comes from the definition of the equivalent permeability: the boundary conditions must keep the true average flow in the considered hydrodynamic cell.

1.1.1 Splitting the Perturbation Fields

In a true oil-gas reservoir with no preferential flow direction the global pressure field may be presented at any time as the superposition of a mean field (constant in space) and a fluctuation field, caused by various external and internal perturbations. The external perturbations are caused by the discrete system of wells, while the internal perturbations are caused by the medium heterogeneity. For the upscaling problem, the physical origin of the perturbations has no great significance; however, another classification of various perturbations can be much more important.

Such a decomposition, which says that all perturbations cause the fluctuations without influencing the macroscale flow, is only valid on the scale of the total reservoir. On the scale of a numerical hydrodynamic cell, which is called the mesoscale and is of the order of a semi-distance between two wells or even smaller, some fluctuations may now produce an oriented flow. For instance, the perturbations caused by the wells certainly determine the mean flow direction within a hydrodynamic cell. Due to this, the pressure field for a hydrodynamic cell can be split into two fields:

$$P(x, y) = P_{dir}(x, y) + P'_{mes}(y) \tag{1}$$

where $P_{dir}(x, y)$ is the directional component of the pressure field, which is caused by the perturbations which determine an oriented average flow within the cell. The term P'_{mes} is the fluctuating component of the perturbation field which causes the mesoscale pressure fluctuations within the cell without determining any oriented average flow. To underline the difference between the scales we have introduced the macroscale space coordinate, x, for the overall reservoir and the local (mesoscale) coordinate, y, for the examined cell, normalized in such a way that the cell length equals 1 in terms of y.

Our idea consists of the following:

- we propose to use only the directional component of the pressure field, $P_{dir}(x, y)$, to formulate the boundary-value conditions for the cell problem;
- we suggest applying analytical methods to calculate the directional field for the overall reservoir;
- we affirm that the fluctuating component, P'_{mes}, can be neglected.

Indeed, let us examine the physical meaning of the directional and fluctuating components. An oriented flow in a cell is caused, in major part, by the system of wells. The fluctuating component is generated by medium heterogeneities of any scale: macroscopic, mesoscopic and microscopic. At the same time, a macroscopic heterogeneity can also influence the average flow in the cell and thus contribute to the directional component. This is, for instance, the case when a macroscale barrier passes near to or across the examined cell. Another object which can significantly influence the average flow direction is the hydraulic macrofracture made near a well. As to the natural macroscale fractures not connected to the wells, they do not significantly deviate the flow, but only accelerate it.

As the objective of the upscaling problem is to obtain the average parameters of the flow, the boundary-value conditions of a cell must be formulated in such a way that the average flow in this cell corresponds to the true flow. From this point of view only the directional component of the pressure field is significant for imposing the boundary-value conditions, while the fluctuating component P'_{mes}, which does not influence the average flow, can be neglected.

So, to formulate the boundary-value conditions it is sufficient to calculate the flow problem for the overall reservoir by only taking into account the system of wells, macrofractures linked to wells, and impermeable barriers. All the other heterogeneities can be ignored. Such a problem in a 2D case for a linear (or linearized) steady-state (or quasi-steady-state) flow equation has analytical solutions. The calculated global directional pressure field, $P_{dir}(x, y)$, will

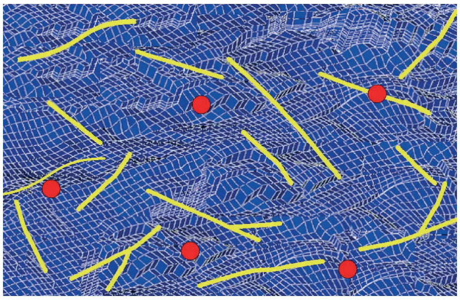


Figure 1

An example of a reservoir perturbed by the wells and the medium heterogeneities.

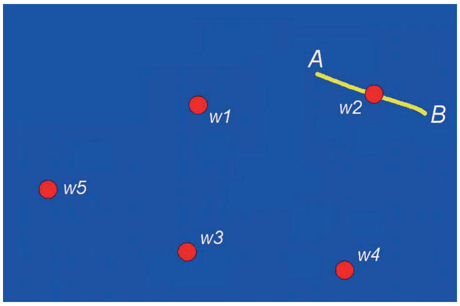


Figure 2

Perturbations determining the directional pressure field.

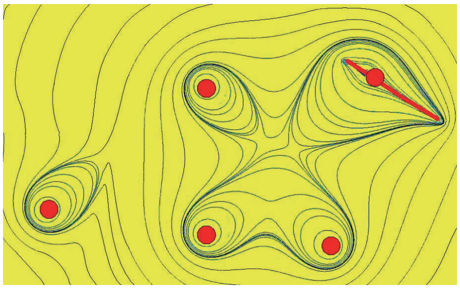


Figure 3

Analytical solution to the directional component of the pressure field.

only be used to formulate the boundary-value conditions for the cell problem for each hydrodynamic cell.

1.1.2 Example of Analytical Calculation of a Directional Component of the Pressure Field

Let us examine the reservoir presented in Figure 1.

According to the suggested method, the directional pressure field corresponds to the perturbation system in a homogeneous reservoir, shown in Figure 2.

The solution to the steady-state problem for the pressure field in such a medium may be obtained by applying the complex potential method:

$$P_{dir} = P^0 + \frac{\mu Q_{fr}}{4\pi k} \ln \left| \frac{x - x^B_{fr} + \sqrt{\left(x - x^A_{fr}\right)^2 + \left(y - y^A_{fr}\right)^2}}{x - x^A_{fr} + \sqrt{\left(x - x^B_{fr}\right)^2 + \left(y - y^B_{fr}\right)^2}} \right|$$

$$+ \frac{\mu}{k} \sum_{i=1}^{N} \frac{Q_{wi}}{8\pi} \ln \left[(x - x_{wi})^2 + (y - y_{wi})^2 \right]$$

where N is the number of wells, and P_0 is the pressure at infinity. The results of simulation based on this equation are shown in Figure 3.

Using these relationships we obtain an analytical formula for the boundary pressure for any hydrodynamic cell.

1.2 Effective Permeability Calculation by the SFMC Method

The second part of the suggested approach is devoted to resolving the cell problem. For a fractured-porous medium we use the basic natural property of such systems: a high difference between the fracture and matrix permeability:

$$\omega \equiv \frac{\langle K \rangle_{\text{matrix}}}{\langle K \rangle_{\text{fracture}}} \ll 1 \tag{2}$$

where the angular brackets mean a simple average.

Using this property, we have developed the method of Splitting the Fracture-Matrix Contributions (SFMC) which consists of the following:

- The contributions of fractures and matrix are split, using the asymptotic properties of a fractured medium.
- The problem of fracture contribution is solved analytically (by the stream configuration method).
- The problem of matrix contribution, being secondary, is solved approximately.

1.2.1 General Definition of Equivalent Permeability

Let $Y=\{-1/2<y_i<1/2,\ i = 1, 2, 3\}$ be the examined hydrodynamic cell, where $y_i = x_i/\varepsilon$ are the local coordinates. We assume that the fractures are filled with a porous medium of a high permeability. So we are dealing with a double porosity medium.

The equivalent absolute permeability, $\widehat{K}_{ik}$, is determined using the result of two-scale homogenization:

$$\widehat{K}_{ik} = \left\langle K(y) \frac{\partial \Psi_k}{\partial y_i} \right\rangle \tag{3}$$

through a family of cell functions, $\Psi_k(y)$, $k=1, 2, 3$, which are the solutions to the following cell problems

$$\frac{\partial}{\partial y_i} \left(K(y) \frac{\partial \Psi_k}{\partial y_i} \right) = 0, \qquad y \in Y \tag{4}$$

Instead of using the periodic boundary conditions for the cell problem (4) as in the case of effective permeability, we use the analytical boundary conditions determined in the precedent section:

$$\Psi_k\Big|_{\partial Y} = \frac{p - \langle p \rangle}{\Delta p_k} + y_k \tag{5}$$

Herein, $\langle p \rangle$ is the average value of the pressure over the unit cell, and Δp_k is the pressure drop in the cell:

$$\Delta p_k = \varepsilon \left(\left| \frac{\partial \langle p \rangle}{\partial x_1} \right| + \left| \frac{\partial \langle p \rangle}{\partial x_2} \right| + \left| \frac{\partial \langle p \rangle}{\partial x_3} \right| \right) \Big/ \operatorname{sign}\left(\frac{\partial \langle p \rangle}{\partial x_k} \right) \tag{6}$$

If there is no local pressure drop along axis x_k, *i.e.* $\partial \langle p \rangle / \partial x_k = 0$, then the periodic conditions must be used for the cell function $\Psi_k - y_k$.

1.2.2 Splitting for a Fractured Medium

Using property (2), we can represent the equivalent permeability as a sum of fractures and matrix contributions separately:

$$\widehat{K}_{ik} = \widehat{K}^{(0)}_{ik} + \omega \widehat{K}^{(1)}_{ik} \tag{7}$$

where $\widehat{K}^{(0)}_{ik}$ is the contribution of the fractures only, and $\widehat{K}^{(1)}_{ik}$ is the contribution of the matrix and of a secondary flow in the fractures:

$$\begin{aligned} \widehat{K}^{(0)}_{ik} &= \int\limits_{Y_f} K \frac{\partial \Psi^f_{k0}}{\partial y_i} \, dV \\ \widehat{K}^{(1)}_{ik} &= \int\limits_{Y_f} K \frac{\partial \Psi^f_{k1}}{\partial y_i} \, dV + \int\limits_{Y_m} K \frac{\partial \Psi^m_{k0}}{\partial y_i} \, dV \end{aligned} \tag{8}$$

The numerical tests show that the contribution of fractures is dominating and, thus, should be calculated exactly. Due to the secondary role of the matrix, the second term in (7) can be calculated in an approximate way:

$$\widehat{K}^{(1)} = \langle K_{\text{matrix}} \rangle \tag{9}$$

1.2.3 Fracture Contribution

To calculate the fracture contribution, we have to solve the following cell problem:

$$\begin{cases} \dfrac{\partial}{\partial y_i} \left(K(y) \dfrac{\partial \Psi^f_{k0}}{\partial y_i} \right) = 0, \qquad y \in Y_f \\ \\ \Psi^f_{k0} = \text{a given function at the boundary} \end{cases} \tag{10}$$

This problem can be solved in an analytical way based on the fact that a thin fracture represents a true streamline. Hence the flow in each fracture is almost mono-dimensional:

$$\Psi^{f(i)}_{k0} = \theta^{(i)}_k \eta + \gamma^{(i)}_k, \;\; k = 1,2; \;\; i = 1,..,N_{\text{segm}} \tag{11}$$

Herein, η is the coordinate along the i-th fracture segment, $Y^{(i)}$; N_{segm} is the number of non-intersecting fracture segments.

This gives us a representation of global flow as a system of streamlines and an explicit representation of the solution along each streamline. By using the Kirchoff conservation law and the continuity conditions in the vertices, cell problem (10) is then reduced to searching for the coefficients $\theta^{(i)}_k$ and $\gamma^{(i)}_k$ from a system of linear algebraic equations:

$$Ax = b \tag{12}$$

The number of equations is equal to a double number of non-intersecting fracture segments.

The existence of the solution was analyzed. We have revealed that the method can formally lead to an overdetermined system, *i.e.* the number of equations is greater than the number of variables. We have strictly proved that the excessive equations are not independent, so the rank of matrix A is always equal to the number of variables. Thus, the problem is well posed.

1.2.4 Matrix Contribution

The contribution of the matrix to the equivalent permeability may be calculated in an approximate way, as this contribution is a secondary effect relative to the role of fractures.

According to our numerical experiments the matrix contribution can be simply approximated by the arithmetic mean value:

$$\widehat{K}^{(1)}_{ik} \approx \langle K_{matrix} \rangle \delta_{ik}$$

This estimation can be improved by taking into account the fluctuations within the matrix only, if we consider the matrix permeability field as low heterogeneous. Then the stochastic averaging yields a corrector term of the order of a square of variance.

1.2.5 Numerical Calculations

The numerical algorithm used to calculate the equivalent permeability of a fractured medium is as follows:

1. Definition of the input data:
   - number of fractures, N_{fr};
   - center positions $(x^{(i)}_c, y^{(i)}_c)$;
   - fracture orientations, $\alpha^{(i)}$;
   - fracture lengths, $l^{(i)}$;
   - fracture apertures, $h^{(i)}$;
   - fracture permeabilities, $K^{(i)}$.
2. Automatic graph fragmentation:
   - construction of the fracture network graph;
   - construction of the set of non-intersected segments.
3. Automatic construction of the matrix A;
4. Solution to the linear system.

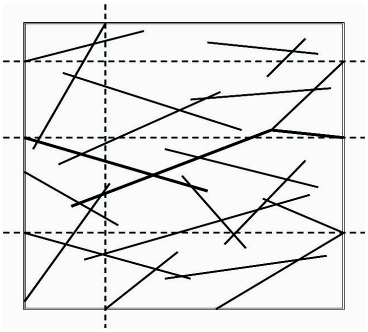


Figure 4

A cell of fractured medium.

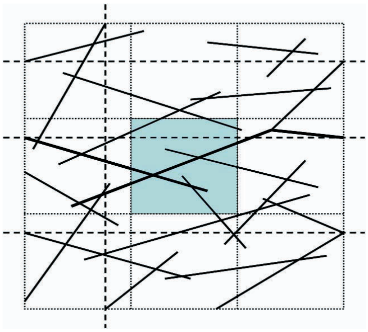


Figure 5

Method of bordering.

To illustrate the SFMC method, we present a numerical test. The fractured unit cell is depicted in Figure 4.

Along with this, we calculated the equivalent permeability (3) by a fine-grid simulation applied to the direct cell problem. For the discretization of this problem we used the finite element method with a grid refinement around each fracture segment. The results of simulations are:

$$\widehat{K}_{ik} = \begin{pmatrix} 32.3 & 1.1 \\ 1.1 & 6 \end{pmatrix}, \quad \text{system of 3678 equations} \tag{13}$$

and by the SFMC method:

$$\widehat{K}_{ik} = \begin{pmatrix} 31.4 & 1.08 \\ 1.08 & 5.7 \end{pmatrix}, \quad \text{system of 82 equations} \tag{14}$$

These two methods yield the same result for the equivalent permeability, but the number of equations used and the used CPU is highly different.

1.2.6 SFMC Method with Bordering

The suggested method can be improved by combination with the bordering method. In order to reduce the error introduced by approximate boundary conditions, we propose displacing the boundary as far as possible from the examined cell. This means that the cell problem will be analyzed within a domain which will include the hydrodynamic cell and all the neighboring cells.

The simulations we made show a low improvement in the equivalent permeability. This confirms implicitly the quantitative correctness and stability of the analytical boundary method suggested in the present paper.

2 HT-SPLITTING FOR COMPOSITIONAL FLOW

We consider an abstract mixture consisting of N chemical species (“components”) able to form two various thermodynamic *phases* separated by an interface from one another. No chemical reactions are analyzed, but each component may be dissolved in both phases. In various thermodynamic conditions, the dissolution degree is variable, which determines a permanent mass exchange in components between two phases. Such a system is frequently called *compositional.*

2.1 Limit Compositional Model for Contrast Properties

The limit compositional model which corresponds to contrast phase mobilities and fast stabilizing relaxation processes was obtained in [13]:

$$0 = \operatorname{div}\left(\psi_g \operatorname{grad} p\right) \tag{15a}$$

$$\left[\frac{\partial \varphi^{(N)}}{\partial \tau} - c_g^{(N)} \frac{\partial \varphi}{\partial \tau}\right] = \frac{\omega}{\varepsilon} \Delta c^{(N)} \operatorname{div}\left(\psi_l \operatorname{grad} p\right) + \frac{1}{\varepsilon} \psi_g \operatorname{grad}\left(c_g^{(N)}\right) \operatorname{grad} p \tag{15b}$$

$$0 = \operatorname{grad} p \left(\frac{\operatorname{grad} c_g^{(k)}}{\Delta c^{(k)}} - \frac{\operatorname{grad} c_g^{(N)}}{\Delta c^{(N)}}\right), k = \overline{1, N-2} \tag{15c}$$

The operations div and grad are performed in dimensionless space coordinates; other definitions are the following:

$$p \equiv \frac{P}{\Delta P}, \qquad \varphi \equiv \frac{\phi \rho}{\langle \phi \rangle \rho_g^0}, \qquad \varphi^{(k)} \equiv \frac{\phi \rho^{(k)}}{\langle \phi \rangle \rho_g^0}$$

$$\psi_i \equiv \frac{\Psi_i k_i(s) \mu_i^0}{\langle K \rangle \rho_i^0} = \frac{K \rho_i \mu_i^0}{\langle K \rangle \rho_i^0 \mu_i}, \ (i{=}g, l), \qquad \tau \equiv t/t_*$$

$$t^* \equiv \frac{L^2 \mu_g^0 \langle \phi \rangle}{\langle K \rangle \Delta P}, \qquad \omega \equiv \frac{\rho_l^0 \mu_g^0}{\rho_g^0 \mu_l^0}, \qquad \varepsilon \equiv \frac{t^*}{t_*} \tag{16}$$

Here superscript “k” refers to the k-th chemical component (k=1,...,N), and indexes g and l to gas and liquid; ϕ is the porosity; $c_g^{(k)}$ or $c_l^{(k)}$ is the mass concentration of the k-th component in gas or liquid $[kg/kg]$, and $\Delta c^{(k)} = c_l^{(k)} - c_g^{(k)}$; s is the volume saturation of pores by liquid; ρ_g and ρ_l are the phase densities; $\rho^{(k)}$ is the partial density of the component k; k_g and k_l are the relative permeabilities; μ is the phase viscosity; K is the absolute permeability; P is the phase pressures; $\Psi_i^{(k)}$ is the mobility of component k in phase i: $\Psi_i^{(k)} \equiv Kk_i\rho_i c_i^{(k)}/\mu_i$.

Parameter t^* is called the global reservoir relaxation time and is equal to the time of propagation of the perturbation caused by a pressure variation, ΔP. Parameter ε is then the ratio of perturbation propagation time to the external process time t_*. Parameter ω is the ratio of the liquid mobility to the gas mobility.

This model is called semi-stationary and can be mathematically derived as an asymptotic procedure based on the small value of the stabilization time regarding the macroscale process time, $\varepsilon \to 0$. In other words, the semi-stationary model can actually be derived as a large-time limit of the compositional model but only if a supplementary parameter of a gas-condensate system is taken into account. This second parameter respects a major property of any gas-condensate system — a much higher mobility of the gaseous phase than that of liquid, $\omega \to 0$.

2.2 Streamline Splitting the Thermodynamics and Hydrodynamics

The last $N-2$ equations (15c) play the basic role in deriving the split thermodynamic model. This sub system may be reduced to time-independent and space-independent differential equations along the streamlines:

$$\frac{d\Phi^{(k)}}{dp} = \frac{1}{\Delta c^{(k)}}\frac{dc_g^{(k)}}{dp} - \frac{1}{\Delta c^{(N)}}\frac{dc_g^{(N)}}{dp} = 0, \quad k = \overline{1, N-2} \tag{17}$$

where

$$\frac{dc_g^{(k)}}{dp} \equiv \frac{\partial c_g^{(k)}}{\partial p} + \sum_{q=1}^{N-2} \frac{\partial c_g^{(k)}}{\partial c_g^{(q)}}\frac{\partial c_g^{(q)}}{\partial p}$$

This means that the full derivative with respect to pressure along a fixed streamline is zero. We thus obtained a system of ordinary differential equations which has a thermodynamic character, as the time and the space variables are not explicitly present in it. This constitutes a principle of HT-splitting (H-hydrodynamics, T-thermodynamics).

2.2.1 Split Thermodynamic Model

Let us unite the basic thermodynamic relations [12] and the limit thermodynamic differential equations (17).

Equilibrium equations for chemical potentials:

$$\nu_g^{(k)}\left(p, \left\{c_g^{(q)}\right\}_{q=1}^{N}\right) = \nu_l^{(k)}\left(p, \left\{c_l^{(q)}\right\}_{q=1}^{N}\right), \overline{1,N} \tag{18a}$$

Equations of phase state:

$$\rho_g = \rho_g\left(p, \left\{c_g^{(q)}\right\}_{q=1}^{N}\right), \rho_l = \rho_l\left(p, \left\{c_l^{(k)}\right\}_{q=1}^{N}\right) \tag{18b}$$

Concentration normalization:

$$\sum_{k=1}^{N} c_g^{(k)} = 1, \qquad \sum_{k=1}^{N} c_l^{(k)} = 1 \tag{19a}$$

Variation of the total composition in an open system:

$$\frac{d\Phi^{(k)}}{dp} = \frac{1}{\Delta c^{(k)}}\frac{dc_g^{(k)}}{dp} = \frac{1}{\Delta c^{(N)}}\frac{dc_g^{(N)}}{dp}, k = \overline{1, N-2} \tag{19b}$$

where the form of the chemical potential functions $\nu_i^{(k)}(p, ...)$ is given.

$2N+2$ equations (18) contain $2N+3$ variables: $c_g^{(k)}$, $c_l^{(k)}$, p, ρ_g and ρ_l. The system is closed if the pressure is given. In this case, this system determines all the concentrations and phase densities as the functions of pressure. Due to this the model is characterized as monovariant.

System (18) represents a limit thermodynamic model for the compositional mixture moving in porous medium. This system is totally independent of the hydrodynamic subsystem. At the same time, this model is valid only along the streamlines. The differential thermodynamic equations (19b) are uniform for any streamline, but the boundary condition for them may be different for various streamlines.

In conclusion, we note that the last $N-2$ equations (15) in the total limit compositional model (15) are transformed into thermodynamic equations (19b) which no longer contain the space and the time variables.

2.2.2 Split Hydrodynamic Model

The problem of gas-condensate flow in terms of the semi-stationary model has the following form:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\psi_g k_g \frac{\partial p}{\partial r}\right) = 0 \tag{20a}$$

$$\overline{\rho}\varphi_l \frac{\partial s}{\partial \tau} + \overline{\rho}\varphi_l s \frac{\partial \zeta_l^{(N)}}{\partial \tau} + \varphi_g(1-s)\frac{\partial \zeta_g^{(N)}}{\partial \tau} = \frac{\omega}{\varepsilon}\frac{1}{r}\frac{\partial}{\partial r}\left(r\psi_l k_l \frac{\partial p}{\partial r}\right) + \frac{1}{\varepsilon}\psi_g k_g \frac{\partial p}{\partial r}\frac{\partial \zeta_g^{(N)}}{\partial r} \tag{20b}$$

with the initial and boundary-value conditions:

$$s|_{\tau=0} = s^0 \tag{20c}$$

$$p|_{r=1} = p_*(\tau) \tag{20d}$$

$$r\psi_g k_g \left.\frac{\partial p}{\partial r}\right|_{r=r_w} = \frac{\varepsilon q(\tau)}{2} \tag{20e}$$

where pressure, $p_*(\tau)$, and the flow rate, $q(\tau)$, are given functions.

Here the functions $\zeta_g^{(k)}$ and $\zeta_l^{(k)}$ are called *the component neutralities* and defined as:

$$d\zeta_l^{(k)} \equiv \frac{1}{\Delta c^{(k)}} dc_l^{(k)}, \qquad d\zeta_g^{(k)} \equiv \frac{1}{\Delta c^{(k)}} dc_g^{(k)} \tag{21}$$

Thus, the total limit compositional model is split into two sub systems of a different origin:

- a closed thermodynamic subsystem (18), which determines all the thermodynamic variables as the functions of pressure along the streamlines;

- a hydrodynamic subsystem (20) consisting of two equations: Equation (20a) describing the spacial pressure field, and Equation (20b) describing the saturation transport; the coefficients of this sub system depend on thermodynamic variables and may then be calculated by using the solution to the thermodynamic subsystem.

2.3 Asymptotic Solution along a Streamline

The split hydrodynamic part reduced to two differential equations may be solved analytically in the case of a 1D flow along streamlines. The method of solution is based on the boundary layer phenomena arising in this system.

2.3.1 Boundary Layer Phenomenon

The problem of gas-liquid compositional flow is characterized by the appearance of a logarithmic fractional-power boundary layer in the domain, which determines a nontrivial saturation behavior. This boundary layer arises in the vicinity of a source or a fracture and is caused by the contrast phase mobilities. This means that the ratio between the liquid and gas mobilities is a small parameter, ω. Due to this the solution can be constructed in the form of an asymptotic expansion with respect to ω. This leads to various asymptotic expansions in various zones. We then develop a singular perturbation method based on matching the asymptotic expansions in these zones.

2.3.2 Asymptotic Solution in the Differential Form

Using the singular perturbation method, we obtained the asymptotic solutions in each zone. The pressure field is invariant with respect to the boundary-value transformation and has the forms:

$$p\,(r,\tau;\varepsilon,\omega) = p_*(\tau) + \varepsilon p_{11}(r,\tau) \tag{22}$$

Here

$$\begin{cases} \dfrac{\partial p_{11}}{\partial r} = \dfrac{q(\tau)}{2r\psi_{g*}k_{g*}} \\[2ex] p_{11}|_{r=1} = 0 \end{cases} \tag{23}$$

Exterior expansion for saturation

The exterior asymptotic solution for the saturation has the form:

$$s = s_* + \varepsilon s_1^{ex} \tag{24}$$

Function s_1^{ex} is obtained from the exterior problem:

$$\frac{\partial s_1^{ex}}{\partial \tau} + A_* s_1^{ex} = \frac{f_{1*}}{r^2} - f_{2*} \ln r \tag{25}$$

with the initial condition: $s_1^{ex}|_{\tau=0} = 0$.

Herein,

$$f_{1*} \equiv \frac{q^2(\tau)\zeta_{gp*}^{(N)\prime}}{4\bar{\rho}\varphi_{l*}\psi_{g*}k_{g*}}$$

$$f_{2*} \equiv \frac{1}{2}\left[s_*\zeta_{lp*}^{(N)\prime} + (1-s_*)\frac{\varphi_{g*}\zeta_{gp*}^{(N)\prime}}{\bar{\rho}\varphi_{l*}}\right]\frac{d}{d\tau}\left(\frac{q(\tau)}{\psi_{g*}k_{g*}}\right)$$
$$+\frac{1}{2}\left[s_*\zeta_{lp*}^{(N)\prime\prime} + (1-s_*)\left(\frac{\varphi_g\zeta_{gp}^{(N)\prime}}{\bar{\rho}\varphi_l}\right)'_{p*}\right]\frac{q(\tau)}{\psi_{g*}k_{g*}}\frac{dp_*}{d\tau}$$

Let us note that here and after index "$*$'' means the value at the domain boundary.

Interior expansion

The relative permeabilities define the structure of the interior expansion. The main property of the relative permeability which significantly influences the structure of the solution and the asymptotic expansions is the behavior in the vicinity of the end-point. According to the percolation theory, in this zone permeability behaves as a power-value function of saturation and has a zero derivative. Therefore the following non-linear relative permeabilities are valid:

$$k_l(s) = \begin{cases} 0, s \leq s_r, \\ \gamma(s-s_r)^\beta, s > s_r, \quad \gamma = const \end{cases} \tag{26}$$

where s_r is the end-point which corresponds to the maximal immobile condensate saturation.

The relative permeability for gas has the form:

$$k_g(s) = 1 - as - O(s^2), \quad a = const \tag{27}$$

Finally, the interior asymptotic solution is:

$$s = s_* + \left(\frac{\varepsilon}{\omega}\right)^{\frac{1}{\beta}} s_1^{in} \tag{28}$$

Function s_1^{in} is obtained from the interior problem:

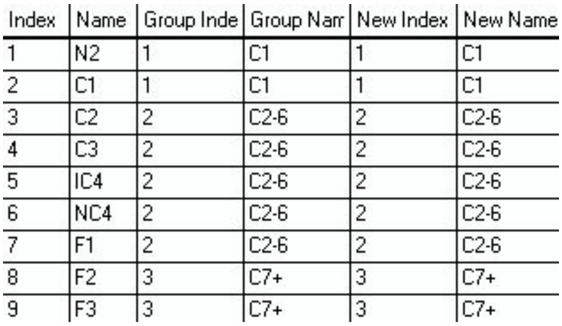
$$\frac{\partial s_1^{in}}{\partial \tau} + A_* s_1^{in} + C_* \frac{\Delta^2}{r}\frac{\partial s_1^{in}}{\partial r} = \frac{f_{1*}\Delta^2}{r^2} \tag{29}$$

where

$$A = \left(\zeta_{lp}^{(N)\prime} - \frac{\varphi_g\zeta_{gp}^{(N)\prime}}{\bar{\rho}\varphi_l}\right)\frac{\partial p}{\partial \tau}, \quad C = -\frac{\psi_l k_l'(s)}{2\bar{\rho}\varphi_l\psi_g k_g}$$

The characteristic size of the boundary layer:

$$\Delta = \varepsilon^{\frac{\beta-1}{2\beta}}\omega^{\frac{1}{2\beta}} \tag{30}$$



Index	Name	Group Inde	Group Nam	New Index	New Name
1	N2	1	C1	1	C1
2	C1	1	C1	1	C1
3	C2	2	C2-6	2	C2-6
4	C3	2	C2-6	2	C2-6
5	IC4	2	C2-6	2	C2-6
6	NC4	2	C2-6	2	C2-6
7	F1	2	C2-6	2	C2-6
8	F2	3	C7+	3	C7+
9	F3	3	C7+	3	C7+

TABLE 1

Mixture composition

2.3.3 Numerical Simulation of the Asymptotic Solution

We shall construct the total asymptotic solutions to the complex multicomponent gas-condensate flow. Let us consider a 9-component mixture, presented in Table 1. The mixture contains light, heavy and neutral components. We consider a gas-condensate reservoir at the initial pressure 325 bars and the temperature 90°C.

A simulator has been developed using the Fortran language, based on the suggested split asymptotic solution for the depletion of any multicomponent gas-condensate reservoir. The next figure shows the results of simulation for the pressure field.

After the initial perturbation the pressure field is fast stabilizing, while remaining a quasi-stationary function. In contrast, the saturation field does not reach any stationary limit and increases with time. The saturations leads to various asymptotic expansions in the characteristic regions with respect to the perturbation parameter and the contrast mobility parameter. Let us use the logarithmic scale for saturation to show such characteristic regions of the fluid displacement mechanism better: Figure 7 for t = 300 days.

As seen, the asymptotic analytical solutions based on the semi-stationary model provide a very fast and adequate description of the reservoir flow. To obtain the same results with the help of the full compositional model, we need much more time for simulation. The ratio of the simulation time for the asymptotic simulator and ECLIPSE is 1:1500. The examined problem takes 2 days of simulation when treated by the full compositional simulator ECLIPSE, whereas the asymptotic simulator takes less then 2 minutes for the same case. This is due to the fact that all the coefficients of the asymptotic diferential equation describing the saturation evolution depend on the boundary pressure only. So this means that the thermodynamic part should be calculated only at the exterior boundary of the domain. Moreover, in the boundary conditions, the fluid behaves usually as a closed thermodynamic system with no flow. Such behavior may be effectively described by a classic thermodynamic model.

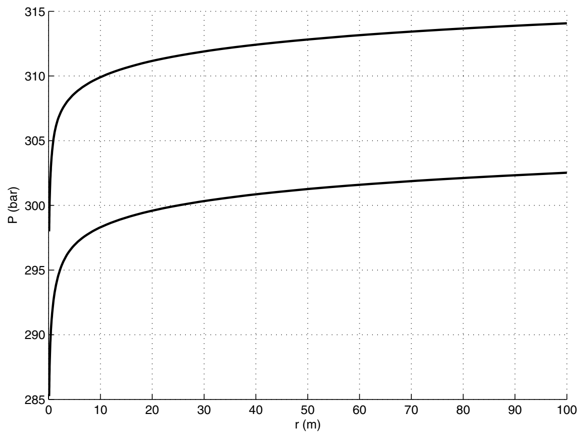


Figure 6

Pressure behavior: a) upper plot for t = 300 days; b) lower plot for t = 1500 days.

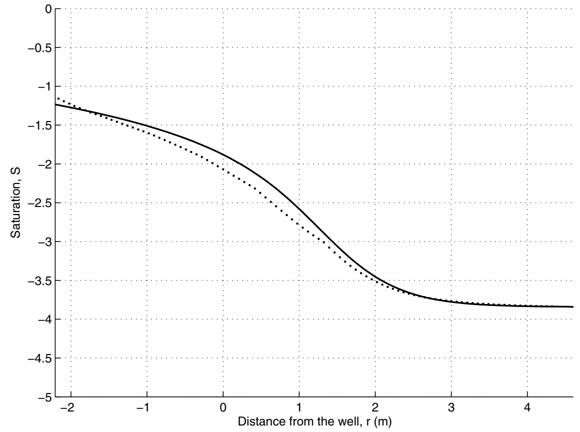


Figure 7

Saturation behavior: a) Eclipse solution (dotted curve); b) asymptotic solution (solid curve).

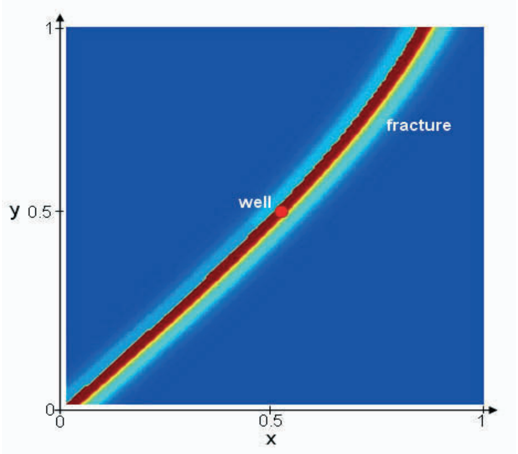


Figure 8

Mechanism of fluid displacement near the fracture.

The other result obtained by using the asymptotic solutions is presented in Figure 8. This is the compositional flow to a fracture calculated using the streamline technique and HT-splitting. We constructed a rather arbitrary field of streamlines oriented to the fracture, by assuming that the fracture plays the role of a discharge, and the streamlines are steady-state. At the boundary of the domain we assumed a constant pressure (or quasi-stationary). The boundary saturation and fluid composition where calculated by using the classic PVT model. For an arbitrary streamline we reformulated the compositional flow model in cartesian coordinates. In the real reservoir pressure remains quasi-stationary relative to saturation, after the short period of stabilization. Due to this the pressure maps can be calculated by the iterative approach using deviation of the estimation.

CONCLUSION

The presented approaches are devoted to hastening reservoir simulation. The first one deals with improving the method of upscaling the permeability field. The second approach is devoted to model compositional flow using the streamline method.

In the section devoted to uspcaling, we suggested two methods:

- a method enabling us to formulate the boundary-value conditions for the cell problem,
- an analytical method of solution for the cell problem for a fractured medium.

In the section dealing with the streamline technique, we developed an asymptotic compositional flow model which ensures a splitting between the thermo- and hydrodynamics. As a result, such a splitting leads to an analytical or semi-analytical solution for the multicomponent flow problem along streamlines.

ACKNOWLEDGEMENTS

The authors would like to express their thanks to the *Schlumberger* Technological Center in Abingdon for financial support of this work.

REFERENCES

1 Bensoussan, A., Lions, J.-L. and Papanicolaou, G. (1978) *Asymptotic analysis for periodic structures*. North-Holland, Amsterdam.

2 Bakhvalov, N.S. and Panasenko, G.P. (1989) Homogenization: Averaging Processes in Periodic Media. Kluwer Academic Publishers, Dordrecht.

3 Sanchez-Palensia, E. (1980) Non-homogeneous media and vibration theory. *Lect. Notes Phys.*, **127**, 129.

4 Edwards, D., Brenner H. and Shapiro M. (1991) Dispersion of inert solutes in spatially periodic two-dimenstional porous media. *Transport Porous Med.*, **6**, 337-358.

5 Edwards, D., Shapiro M. and Brenner H. (1992) Dispersion and reaction in two-dimensional model porous media. *Phys. Fluids A*, **5**, 837-848.

6 Neuweiler, I., Attinger S., Kinzelbach W. and King P. (2003) Large scale mixing for immiscible dipslacement in heterogeneous porous media. *Transport Porous Med.*, **51**, 287-314.

7 Panfilov, M. (2000) *Macroscale Models of Flow through Highly Heterogeneous Porous Media*. Kluwer Academic Publishers, Dordrecht.

8 Panfilov, M. and Floriat S. (2004) Nonlinear two-phase mixing in heterogeneous porous media. *Transport Porous Med.*, **57**, 347-375.

9 Tran, P. (2000) Corrections non linéraires à la loi de Darcy et dispersion en milieux porex périodiques anisotropes. *PhD. thesis*, Univeristé Paris 6.

10 Wen, X. and Gomez-Hernandez J. (1996) Upscaling hydraulic conductivities in heterogeneous media: an overview. *J. Hydrol.*, **183**, 9-32.

11 Wu, X. and Hou T. (2002) Analysis of upscaling absolute permeability. *Discrete Cont. Dyn. B*, **2**, 185-204.

12 Danesh, A (1998) *PVT and phase behaviour of pertoleum reservoir fluid*. Elsevier Science.

13 Oladyshkin, S., and Panfilov, M. Two-phase flow with phase transitions in porous media: instability of stationary solutions and a semi-stationary model. *Third Biot Conference on Poromechanics*. Norman, Oklahoma, USA.

Final manuscript received in December 2006