

# Recent Advances in Upgridding

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**Résumé — Récents progrès en Upgridding** — L'upscaling (ou changement d'échelle) est le processus par lequel les propriétés d'une description géologique à haute résolution sont approchées par un modèle de simulation à basse résolution. L'upgridding est le processus par lequel nous déterminons la résolution spatiale (la grille numérique 3D) du modèle de simulation. Dans nos travaux en cours, nous explorons diverses mesures de l'erreur introduite par les approximations du changement d'échelle. Ces mesures sont basées sur les propriétés de flux local et, dès lors, ne dépendent pas de la connaissance des modes d'écoulement globaux du fluide. Se basant sur ces mesures d'erreur, il est possible de concevoir une grille numérique qui soit optimale, c'est-à-dire qui introduise le changement d'échelle minimal pour un nombre spécifique de cellules.

Nous explorons trois applications différentes de ces mesures d'erreur. Dans la première, un schéma stratifié d'une simulation d'un réservoir optimal est déduit du schéma stratifié de la description géologique par groupage séquentiel des strates afin de minimiser l'erreur. L'erreur associée à la séquence explore les courbes d'options entre la distorsion et la variance, le schéma optimal étant déterminé par un équilibre entre les deux.

La seconde application se situe dans le domaine du changement d'échelle en surface. Nous utilisons ici deux définitions différentes de la transmissibilité et des données croisées dans le changement d'échelle du tenseur de perméabilité afin de déterminer des mesures locales de l'erreur.

L'application finale examine d'autres contraintes ou considérations que nous pouvons appliquer sur la forme de la grille de simulation du réservoir. En terme de caractérisation du réservoir, ces contraintes sont associées à la préservation de la structure du gisement, à la stratigraphie et aux couches ou, localement, à la continuité du faciès des roches. La simulation et les procédés de représentation auront aussi un impact sur la conception de la grille optimale, c'est-à-dire que la conception de la grille de simulation optimale diffèrera entre un procédé d'injection d'eau et un procédé d'injection de gaz. La grille de simulation résultante sera un composé de cellules point-coin avec la plupart des avantages numériques d'un maillage déstructuré de type **PEBI**, sans la nécessité de revoir notre technologie de simulation pour utiliser de telles grilles.

**Abstract — Recent Advances in Upgridding** — Upscaling is the process whereby the properties of a high resolution geologic description are approximated by a lower resolution flow simulation model. Upgridding is the process whereby we determine the spatial resolution (the 3D computational grid) of the simulation model. In our current work we explore different measures of the error introduced by the upscaling approximations. These measures are based upon local flow properties and hence are not dependent upon knowledge of global fluid flow patterns. Based upon these error measures it is possible to design a computational grid that is optimal, i.e., that introduces the minimal upscaling error for a specific number of cells.

We explore three different applications of these error measures. In the first, an optimal reservoir simulation layering scheme is derived from the layering scheme of the geologic description by sequentially grouping the layers in such a way to minimize the error. The error associated with this sequence explores the trade-off between upscaling bias and variance, with the optimal scheme being determined by a balance between the two.

The second application is in areal upscaling. Here we utilize two different definitions of transmissibility and the cross-terms in the upscaled full permeability tensor, to provide local measures of error.

The final application examines other constraints or considerations that we may place upon the reservoir simulation grid design. In terms of reservoir characterization, these constraints are associated with preserving reservoir structure, stratigraphy and layering, or locally, facies continuity. Simulation and process representation will also have an impact on the optimal grid design, i.e., the optimal simulation grid design will differ between a waterflood and a gas displacement process. The resulting simulation grid will be a composite of corner point cells, with many of the computational advantages of an unstructured PEBI grid, without the requirement to revise our modeling technology to utilize such grids.

## INTRODUCTION

Much of the work in our industry is based upon simple grouping concepts. For instance, whether discussing reservoir zonation (a geologist) or flow units (a reservoir engineer) the intent is to find those portions of a reservoir with similar static or dynamic characteristics. The description of the reservoir starts from the simplest choice of layers and units and then adds detail and increased resolution until diminishing returns are obtained. The work of Testerman (1962) provides an example in which statistical techniques of “within layer” and “between layer” variance are utilized to guide this sequential refinement. This describes how a traditional reservoir characterization study may have been performed.

How does this differ from how a reservoir characterization study may be performed today? In the traditional approach, detail is sequentially added to a simple description. In contrast, a modern reservoir characterization study, based upon a detailed 3D geologic description, provides us with the opportunity to either retain or remove detail, as required. Upscaling is the process whereby we suitably average the reservoir properties as we remove detail. Upgridding is the process whereby we choose the appropriate resolution for a calculation. It can be thought of as the complement to the traditional approach of “down-gridding”, or of adding detail to a simple description.

What is an appropriate resolution for a calculation? Of course, there is no single answer because there is no single purpose for a calculation. Whether we are performing a simple screening calculation, a pressure history match, a near well study, a mechanistic evaluation of a miscible flood or a history match of a mature waterflood with more than a decade of data, the scale at which we need to work will vary tremendously. None the less, with the foundation of a detailed 3D geologic description, we will show several examples in which an analysis of the description will guide us to an optimal appropriate grid resolution based upon an error analysis.

Our specific approach will differ from earlier studies, for instance, by Durlofsky *et al.* (1996), and Stern and Dawson (1999) in that we will utilize an *a priori* local error analysis. Specifically, we will calculate the error in upscaling as a sum of local errors, where these errors are obtained without knowledge of well locations, well rates, and reservoir flooding patterns. The techniques of Durlofsky and Stern each rely upon a choice of “reasonable” sweep patterns and calculate an error based on global flood response. Their upgridding calculations are based on global *a posteriori* error estimates where in contrast ours are local *a priori* estimates. In some sense our approach is less optimal for a given placement of wells but at the same time we expect the results to be more generally applicable. Our layer grouping approach is similar to the work of Li *et al.* (1995) and of Li and Beckner (2000), which are also based upon local *a priori* error estimates. Our approach differs from their work in that we use a more physically motivated approach to the error analysis with a better capability to derive an optimal layering scheme with good reservoir performance prediction.

This paper consists of a number of case studies to exemplify different error assessments and different upgridding applications. Examples of the *a priori* errors introduced while upscaling are summarized in Table 1. In each case, the analysis is based upon the “missing physics” in the upscaled coarsened model. In each case, we calculate upscaling errors based on features of the fine scale reservoir model or fluid flow which are not present in the coarsened model. In the first case study the errors will be primarily geometric in which we lose, for instance, resolution of reservoir juxtaposition across faults, or resolution of fluid contacts, as a model is coarsened. The second set of case studies is based upon a calculation of multiphase frontal velocity variation within a coarsened cell. Of course, only a single velocity can be represented within a coarse cell, but on the fine scale a velocity distribution is certainly present. These case studies are used to design a coarse reservoir layering scheme given a high

resolution reservoir description. The last case studies utilize local error estimates in upscaling to design locally variable simulation grids. The latter have many of the advantages of unstructured PEBI grids whilst retaining the integrated project workflow and numerical advantages of an underlying regular gridding structure. Many of these case studies appeared in King *et al.* (2006), but we expand upon those discussions in terms of a generalized approach to *a priori* error estimates for developing appropriate coarsened models.

TABLE 1

Summary of *a priori* errors introduced while upscaling

Upscaling example	<i>A priori</i> error: missing physics
Layering grouping: 1 × 1 × N upscaling	Multiphase frontal speed
Vertical upscaling	Pay / non-pay facies contrast
Grid coarsening	Connectivity, especially at faults
Saturation	Fluid contacts
Transmissibility upscaling: areal upscaling	Full tensor permeability

### 1 CONSTRAINED VOLUMETRIC COARSENING

We start with a pre-production case study of a faulted turbidite reservoir for which a number of detailed reservoir simulation models have already been built. We wish to explore a wide range of performance predictions to help design the early field life surveillance program. For these screening studies the target was to reduce the simulation run times by an order of magnitude and to introduce no more than a 10% error in cumulative oil production. An example coarsened simulation grid is shown in Figure 1 and the results of this coarsening study are shown in Table 2. Each entry in the table represents a “rule based” upgridding calculation in which different aspects of the fine grid model were retained in the coarsened computational grid. The two reference calculations are the first and the last in the table. The first entry is the fine scale simulation model, with a cumulative oil error of 0%, a cpu time ratio and an active cell ratio each of 100%. The last entry is that of a manual rebuild of this model at a coarsening ratio of 3 × 3 × 3. This included a reconstruction of a coarser version of the structural model, layering scheme, simulation grid, and upscaled properties. Including an error analysis and additional pseudoization, the rebuild of the model occupied six weeks of the screening study. Our intent was to match the results of this earlier screening work in far less time while at the same time exploring the aspects of the reservoir description which were most important to preserve. Figure 1 corresponds to the next to last case, labelled “Selective: Aquifer + Wells” in Table 2. We will describe each of these cases in turn.

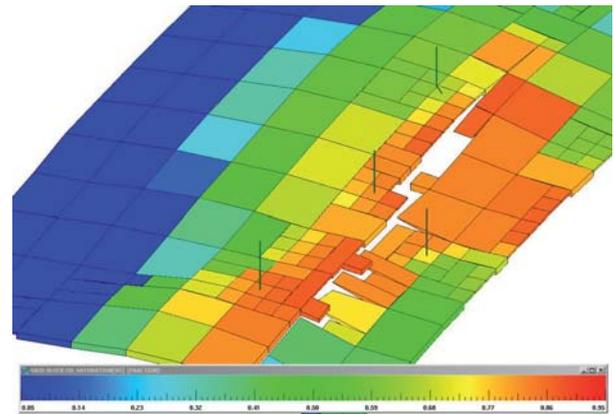


Figure 1

Three dimensional unstructured simulation grid resulting from volumetric coarsening, for the case “Selective: Aquifer + Wells” of Table 2. The color scale represents initial water saturation.

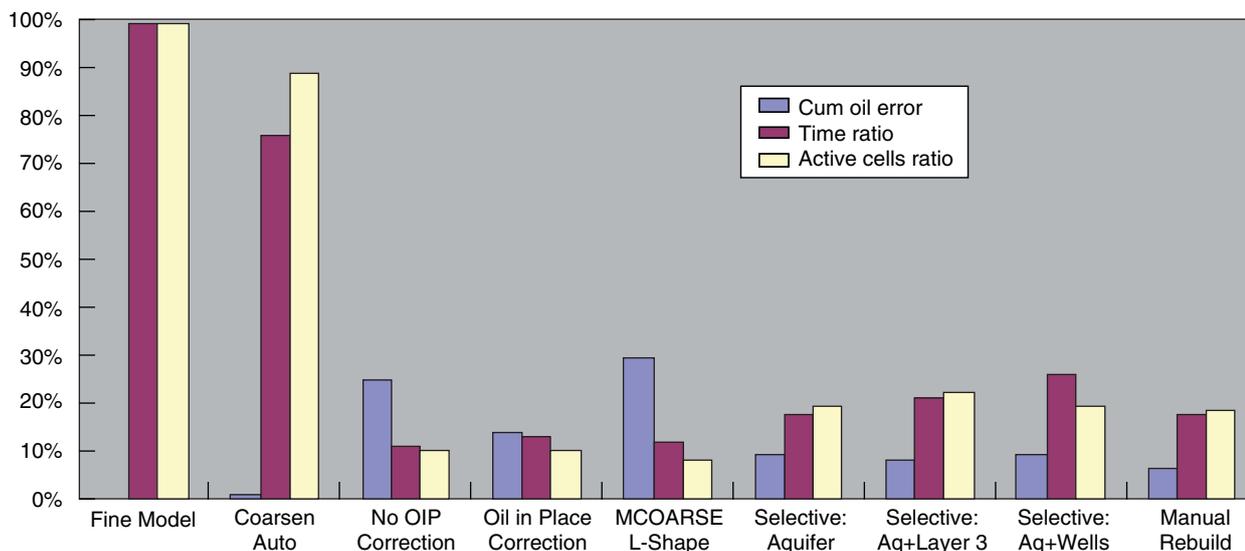
We first utilized the “Coarsen Auto” feature of the VIP reservoir simulator (2003) with a coarsening ratio of 3 × 3 × 3. With this feature if any cells are faulted within the coarsening volume then that portion of the model is retained at fine resolution. With this constraint we see that only ~10% of the active cells are removed from the model with a reduction in run time of only ~20%. Although the resulting simulation provides an excellent approximation to the original model it does not provide the required screening tool.

The grid of the second study was our first attempt to develop a flexible gridding strategy near faulted cells. If it was not possible to obtain a 3 × 3 × 3 coarsened cell without merging different fault blocks then other rectangular coarsening ratios were applied locally. This strategy reduced the cpu time ratio and active cell ratio to our desired target. In fact, the run time reduction was superior to that of the manual rebuild and no pseudoization was necessary. Unfortunately, there was close to a 25% error in calculated cumulative oil.

We had available the insight from the pseudoization study of the manual rebuild. This changed our emphasis from the reservoir structure to the reservoir fluids. With the 3-fold loss of lateral resolution within a simulation layer, cells that were cleanly on either side of the original oil water contact now straddled the contact. The resulting original oil in place calculation was less accurate and this was corrected for in the “Oil in Place Correction” example. We used exactly the same grid as in the previous case but now with the original oil in place calculated on the fine grid and then upscaled to this coarser grid. This supplied a case that was close to our run time and accuracy targets and would have been sufficient for our screening studies. None the less, we continued to explore other coarsening strategies.

TABLE 2

Summary of cases studied using three dimensional unstructured volumetric coarsening, as in Figure 1



1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24

1	1	1	1	5	5
1	1	1	5	5	5
1	1	1	5	5	5
1	1	5	5	5	5

Figure 2

Representation of a non-rectangular coarsening using the VIP MCOARSE array construction. The 6 × 4 24 cell fine scale model is coarsened into an effective 2 cell model with the values shown in the MCOARSE array.

The next approach identified a technical limitation in our simulation upscaling approximations. We recognized that restricting our approach to *rectangular* coarsening could easily generate 27:1 bulk rock volume contrasts in adjacent cells, which we expected to negatively impact the simulation run times. 9:1 areal contrasts can be found in Figure 1. In 3D, 27:1 contrasts can arise. The MCOARSE flexible coarsening capability of VIP (2003) allowed us to represent the rectangular cells of Figure 1, but it also supports a more general gridding structure, Figure 2, in which small cells could be

merged with adjacent larger cells. This removes the large cell volume contrasts but it introduces non-rectangular coarsened corner point cells into the simulation grid. Figure 2 is perhaps the simplest example, which, on a coarse grid would be easy to approximate as a tilted corner point cell. However, even for this simple a case, how should the simulator calculate the transmissibility from Cell 1 to Cell 5? On the fine scale, the flux is a sum across 6 faces in two different coordinate directions. It is not obvious how we can use a simple algebraic approximation to obtain the coarse cell flux. Instead it should be approached as a formal upscaling calculation for the transmissibility on the coarse simulation grid. This remains an active area of research. Interestingly, for this example, the accuracy of the calculation was reduced but with negligible impact on the run time. The pore volume contrast between cell volumes was not impacting the simulation run time. Instead it was controlled by the many non-neighbor connections across the faults in the model.

We continued to develop strategies to reduce the error in cumulative oil, based on rectangular coarsened cells. The next two cases introduced two additional constraints. In “Selective: Aquifer” cells within the oil column were no longer coarsened laterally. This almost doubled the number of cells compared to the “OIP Correction” case and also increased the run time, but provided an improved estimate of the cumulative oil. This recognizes that not only do we lose resolution of fluids volumes when we coarsen across fluid contacts, but that we also lose accuracy in the calculation of phase relative permeabilities. A similar effect could have been achieved on the coarser grid of the “OIP Correction”

case if we had invoked vertical segregation of fluids in the calculation of the relative permeabilities in the simulator.

Examination of the aquifer progression in this model showed one major remaining discrepancy between it and the fine scale simulation. The fine scale model included a major flow barrier in “Layer 3” of the reservoir description. We now treated this layer as an additional constraint in that no vertical coarsening was permitted across that layer. The results are shown in “Selective: Aquifer + Layer 3”. This case has the minimum error in calculated cumulative oil of all the cases studied, although it is almost twice as expensive as the “OIP Correction” case. Either of these two cases would be suitable for the screening study.

The last case studied did not include Layer 3 as a gridding constraint. Instead, resolution was increased near each well to match the fine grid. This is the grid shown in Figure 1. Interestingly, the error in cumulative oil was not reduced, although this did increase the cpu run time by about 50%. Perhaps this was not a surprise, but it did point out that an increase in local resolution did not compensate for lack of resolution between wells.

It is also worth noting the workflow implications of these calculations. Unlike the usual project workflow in which an explicit simulation grid and properties must be developed from the geologic model *before* simulation, all of these calculations were performed with the grid and properties of the geologic model supplied to the simulator. The MCOARSE array then invokes run-time upscaling calculations *within* the simulator to develop an upgridded simulation grid with effective properties which is then simulated. Change of simulation resolution is extremely simple as it consists of no more than replacing one MCOARSE array with another. This also benefits the workflows within a team as the geologic model then becomes the “shared earth model” for both the reservoir engineer and the geologist.

What conclusions can we draw from this study? For a screening calculation we found, in decreasing order of importance, that the following constraints were important when designing a rule based coarsened simulation grid:

- resolution of fault block boundaries, and flow across faults;
- resolution of fluid contacts;
- resolution of major stratigraphic zone boundaries.

These three resolution requirements provide the components of a local *a priori* error estimate that may be used generally to guide the design of a spatially variable computation grid. We recognized that increasing resolution only near the wells was not sufficient to improve the accuracy of the calculation. Finally, we also identified technical issues in the accurate calculation of transmissibility for non-rectangular coarsened cells. This last is an example of an *a priori* numerical error which will also constrain the design of the coarsened grid.

## 2 LAYER GROUPING

This upscaling error analysis is based on a  $1 \times 1 \times N$  vertical coarsening of the cells of a high resolution 3D geologic or reservoir simulation model. The result of the analysis is a recommended layering scheme – a list of the  $N$ -fold coarsenings, where  $N$  is now variable within the model to better capture the internal reservoir heterogeneity. The error analysis (Table 1, line 1) is based upon the observation that even with a “perfect” single phase upscaling calculation, that a distribution of multiphase frontal velocities is replaced with a single value. This is sketched in Figure 3 for a three layer system. Each layer has a local interstitial speed proportional to, let’s say,  $(k_x/\phi)$  and this distribution is replaced by a single value of  $(k_x^{Eff}/\phi^{Eff})$  based on the effective properties of the coarse cell. The “missing physics” in a single phase flow calculation is the multiphase frontal speed. As discussed in King *et al.* (2006) we have used the net rock volume weighted variance of the distribution of  $(k_x/\phi)$  about its mean as a measure of the error introduced in this upscaling calculation. Specifically, we perform a sequential coarsening calculation, based on merging the layer pair which minimizes the increase of variation within the coarse cell. We can utilize a fast recursive calculation to analyze even a large geologic model in a few minutes.

The results of a calculation are shown in Figure 4. The calculation begins with a high resolution fine scale model, with the “Within Cell” variation equal to zero for each cell. As each layer is merged in turn, this within cell variation is increased depending upon the variance of the local interstitial velocity distribution. The sequence is repeated in turn until a single layer model is obtained where the within cell variation is a maximum for each cell. The within cell variation is summed to determine the total variation (or heterogeneity) of

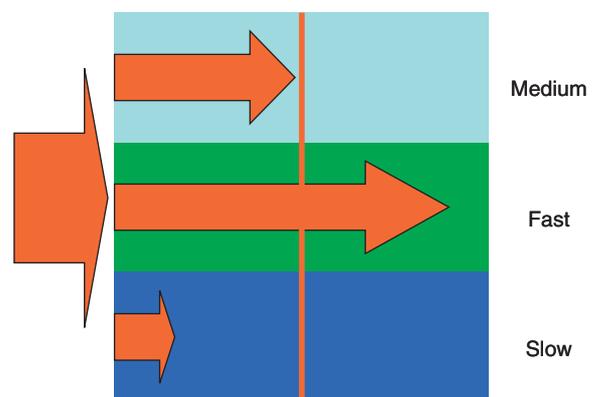


Figure 3

A distribution of multiphase frontal velocities is replaced by a single frontal velocity upon coarsening.

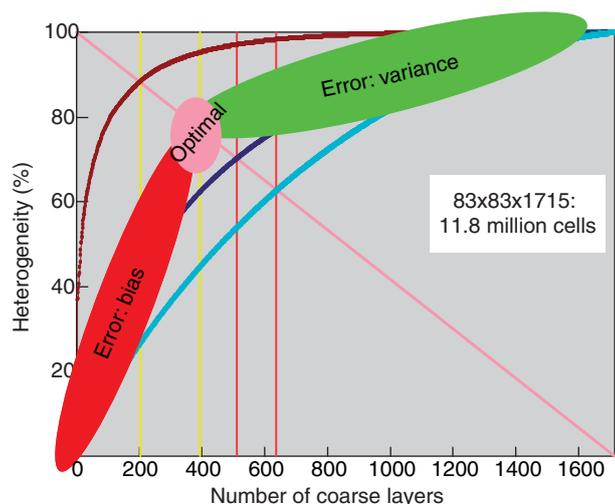


Figure 4  
Layer grouping error analysis.

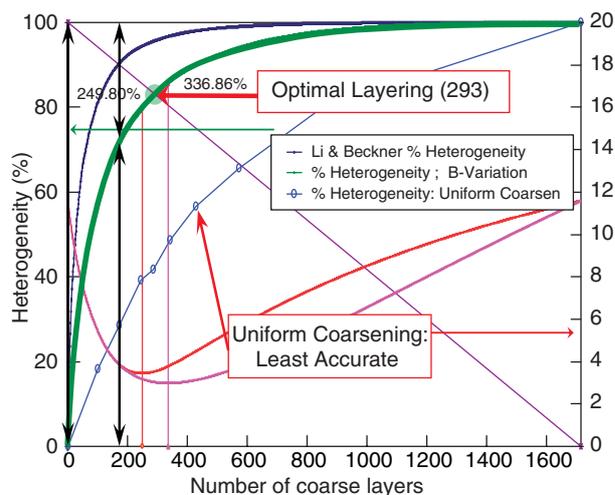


Figure 5  
Tight gas layer coarsening example.

the model. Knowing this total variation, the characteristic curves of Figure 4 can then be plotted. As shown in the Figure, there are two trends in these curves. Along the top of the plot is the region marked “Error: Variance”. Here as layers are grouped and the explicit heterogeneity is removed from the model, the variance is gradually reduced. In contrast along the left side of the model, once a certain degree of heterogeneity is removed, then removing any more begins to rapidly reduce the amount of heterogeneity represented in the model. In this case, the fundamental description is being over-homogenized and the performance prediction will be biased towards that of a homogeneous reservoir. The optimal layering scheme, in that it utilizes a minimum number of layers but preserves sufficient detail not to be biased, is the region on the curve with maximum curvature.

Let’s examine the case study for a 1715 layer tight gas reservoir study in Figure 5. The thick curve is the result of the calculation just described. Two auxiliary curves are derived from this curve and are shown at the bottom of the Figure. They are plotted on the right hand axis. They are used to bracket the region of maximum curvature which in this case runs from 249 layers and 80% heterogeneity to 336 layers and 86%. We choose as the optimal the center of this interval, which is at 293 layers. In short, the thick curve shows how much heterogeneity is preserved (83% for the optimal layering scheme) and how much has been lost (17% in this case).

The most interesting remaining curve in this Figure is marked “Uniform Coarsening”. This curve characterizes the usual approach of uniform layer coarsening. A number of cases have been generated ( $1 \times 1 \times 2$ ,  $1 \times 1 \times 3$ ,  $1 \times 1 \times 4$ , etc.) and for each the heterogeneity has been calculated. For instance,  $1 \times 1 \times 2$  coarsening preserves 80% of the heterogeneity,  $1 \times 1 \times 3$  coarsening preserves 65%, and so on.

A 285 layer model ( $1 \times 1 \times 6$  coarsening) preserves only a little more than 40% of the heterogeneity, compared to 83% for the optimal layering scheme. In practice the run times for the two computations are comparable, but the optimal layering scheme provides a more accurate result.

The simulation results are shown in Figure 6. Here the cumulative gas recovery at end of project life has been calculated for the fine scale model, to use as a reference (dashed line), and for each of the coarsening cases. The optimal layering scheme is marked with a triangle and provides a very good approximation to the fine scale result. Other layering schemes have also been explored and are included in this plot to show the general trend. For models coarser than the optimal choice, the cumulative gas recovery diverges from the fine scale result. The distinction between variance reduction and bias increase is seen in these results.

The simulation results for uniform coarsening are also shown in Figure 6. This is the curve that deviates the most from the fine grid result. For small degrees of upscaling ( $1 \times 1 \times 2$ ,  $1 \times 1 \times 3$ ,  $1 \times 1 \times 4$ ) the cumulative gas recovery is gradually reduced. The upscaling calculation utilizes a simple arithmetic average for the horizontal permeability and a harmonic average for the vertical permeability. As the coarse cell thickness increases, increasing amounts of pay and non-pay will merge, reducing the vertical communication in the model. However, once the coarse cells are sufficiently thick then a single coarse cell will consist of what would have been disconnected pay on the fine scale. In this case the cross-sectional sweep will be artificially increased. This is what causes the rapid increase in calculated gas recovery for uniform coarsening ratios greater than  $1 \times 1 \times 4$ . We will return to this discussion when describing unstructured vertical coarsening, in the next section of this paper.

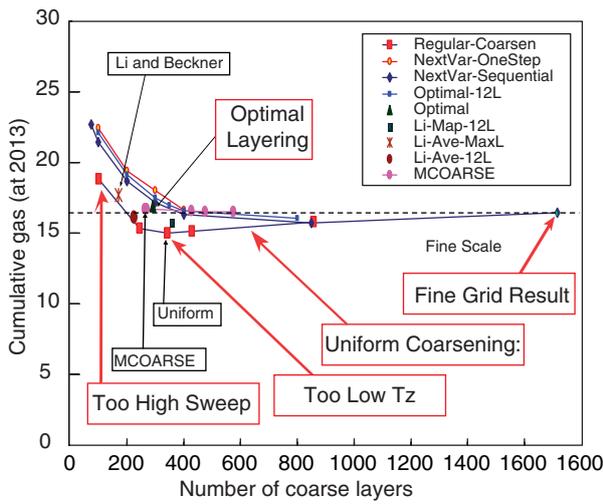


Figure 6  
Tight gas cumulative recovery.

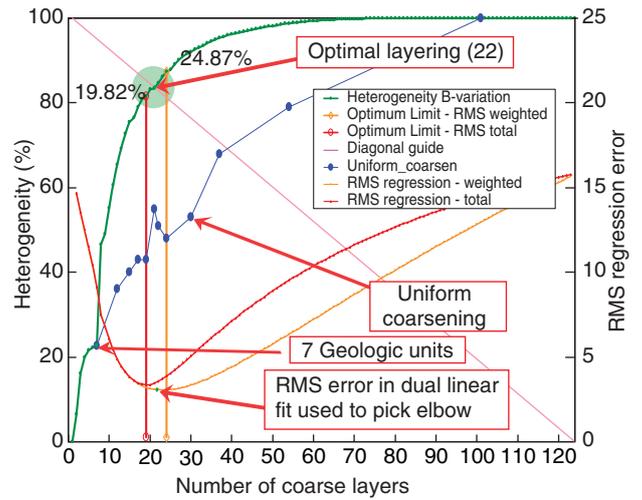


Figure 7  
Waterflood layer grouping analysis.

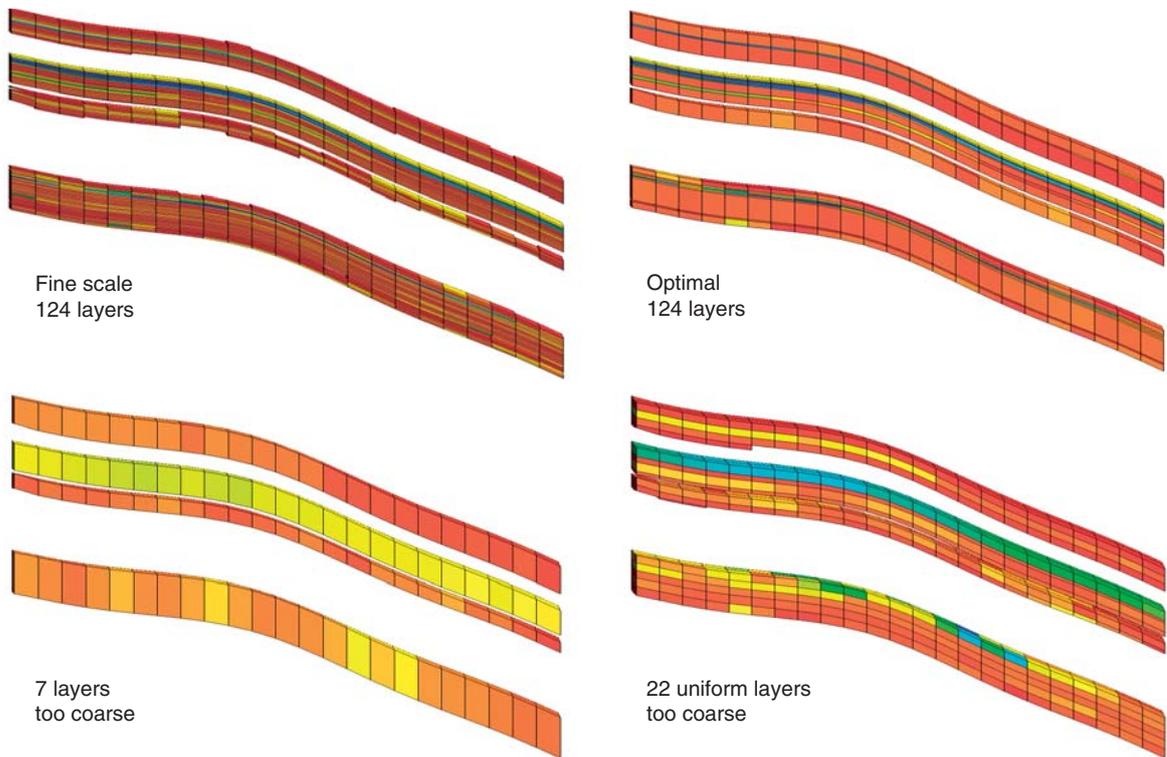


Figure 8  
Waterflood layer groupings.

The tight gas example is an example of primary depletion. The upgridding calculation is dominated by the requirement to preserve the continuity within the geologic model. How well does this analysis apply for waterfloods? Figure 7 shows the layer grouping analysis for an onshore waterflood, in this

case, reducing a 124 layer geologic model to an optimal simulation model. The optimal layering of 22 layers preserves 85% of the heterogeneity. In contrast, a uniform coarsened result, also at 22 layers, preserves approximately 50% of the heterogeneity. Notice that because the uniform

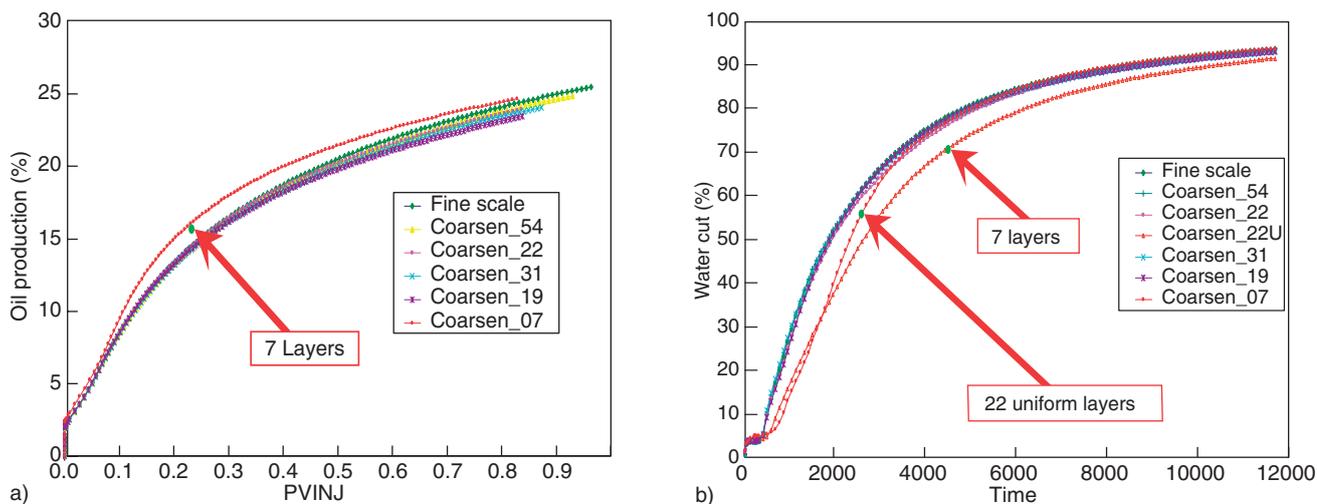


Figure 9

Waterflood performance prediction.

coarsening is not sequential, you can sometimes “get lucky” as in this case, where the amount of heterogeneity preserved increases as the number of layers decreases. Good practice for this analysis treats the geologic units discretely and so the coarsest simulation, in this case, has 7 layers: one for each geologic unit.

The resulting optimal 22 layer scheme is shown in Figure 8. Also shown are the fine scale model (124 layers), the coarsest simulation model (7 layers) and a uniform 22 layer model. The optimal layering scheme preserves high permeability streaks and flow barriers in a way that the uniform 22 layer scheme does not.

Figure 9 shows the predicted oil production for the coarsened cases with 54, 31, 22, 19 and 7 non-uniform layers. In these calculations there is a slow decrease in oil production rate away from the fine scale result, consistent with the discussion we have already had of decreased vertical permeability for thicker upscaled sections. However, the 7 layer case is definitely too coarse, and generates the outlier in the oil production curve. The water cut prediction includes these curves and the 22 layer uniform result. Now both the uniform 22 layer result and the 7 layer result deviate from the fine scale results which are preserved by the non-uniform layering cases.

What have these case studies demonstrated? We have shown that an analysis based on “missing physics” can be used to guide us to the best layering scheme for a specific number of layers. Perhaps more importantly, it can predict when a layering scheme is too coarse and no longer provides a good representation of the reservoir heterogeneity. It is, of course, possible to decrease the number of layers beyond the optimal number. In such a case, the error analysis would indicate that an over-homogenized biased solution would arise. Other techniques, such as pseudoization, would then be necessary to compensate.

### 3 VOLUMETRIC COARSENING, REVISITED

#### 3.1 Unstructured Vertical Coarsening

In the previous section we have seen how an error analysis can be used to derive a coarse reservoir simulation layering scheme. It was certainly technically successful, but can we do better? Returning to Table 1 for our catalogue of errors, we have seen how to use the variation in the multiphase frontal speeds to calculate an upscaling error. We have also seen how, even for the optimal layering schemes, pay and non-pay are mixed within single coarse cells providing too low an estimate of the vertical permeability. This error is listed in line 2 of the table. How can it be managed?

We have already indicated that the geological unit is treated as a strong constraint when coarsening. For instance, for the waterflood example, the coarsest simulation model had 7 layers. We can generalize the concept of a strong constraint to prevent pay and non-pay from merging. In such a case, the vertical permeability will never be artificially reduced to zero. In addition, sands from distinct pay intervals will never be merged. Of course, to do this effectively, we must also allow each column of the model to be layered independently, generating a totally unstructured numerical problem.

The grid that results from this approach for the tight gas example studied previously is shown in Figure 10. This Figure also includes a portion of the 3D geologic model. This is a low net to gross fluvial reservoir but with sufficient sand continuity in three dimensions to have an extremely successful project. The cross section shown in the Figure shows only the active cells in the simulator. The non-pay cells are inactive and have been totally removed from the calculation. The cells are colored by pressure, and range in value from 2000 to 7000 psi. Clearly the unstructured computational grid has

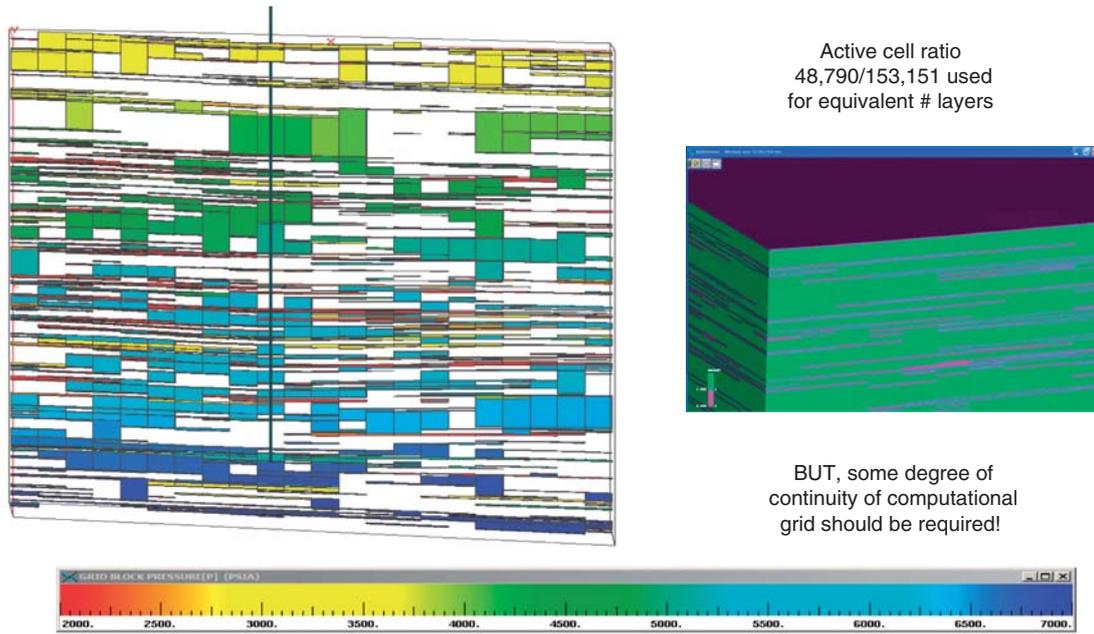


Figure 10  
Tight gas unstructured vertical coarsening.

been able to preserve both the connectivity and the disconnections of pay of the geologic model. The results of the flow simulation are shown in Figure 6, for four models with increasing amounts of coarsening. The cell by cell within variation, computed earlier, was used to merge cells similarly to how we had merged layers earlier. Now that we have a strong constraint, all of the cases are essentially identical to the fine scale result. In addition, the coarsest possible case (the leftmost case) provides as good a performance prediction as the fine scale model.

### 3.2 Unstructured Areal Coarsening

The intent of an upscaling calculation is to reduce simulation run times. The result of an upgridding analysis is to ensure that the resulting simulation run is as true to the underlying geologic description as possible. In the previous sections we have emphasized vertical upscaling as we typically upscale over large intervals vertically where an error analysis can be very informative. Areal coarsening tends to be over extremely small ratios:  $2 \times 2$  coarsening is probably the most common. What can an error analysis teach us here?

Returning to the last line of Table 1 we are reminded that a conventional simulator uses a two-point transmissibility construction, which, roughly speaking, assumes that the flux and the pressure gradient from cell to cell are parallel. Specifically, there are no flux components driven by transverse pressure gradients. Unfortunately, such cross-terms arise

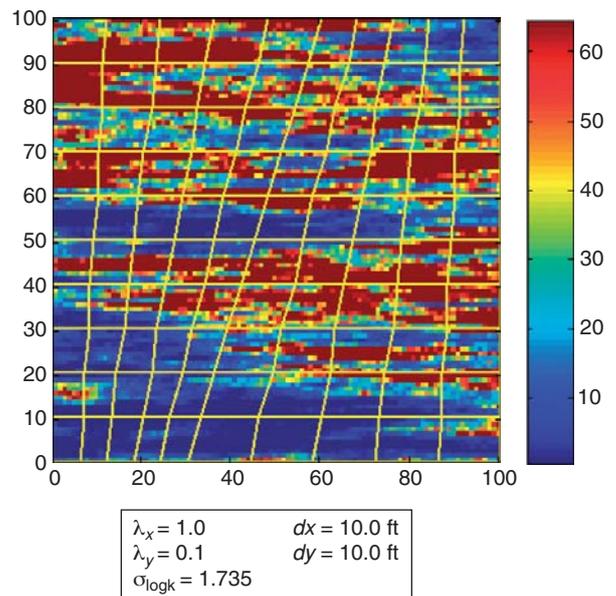


Figure 11  
Fine scale areal cross-section with coarse corner point grid.

naturally in channel systems where, in general, the channels are not aligned with the local axes of the coarse grid. In such a case, we can perform a full tensor permeability upscaling calculation to determine the magnitude of the cross-terms, to judge where the two-point flux approximation is not accurate.

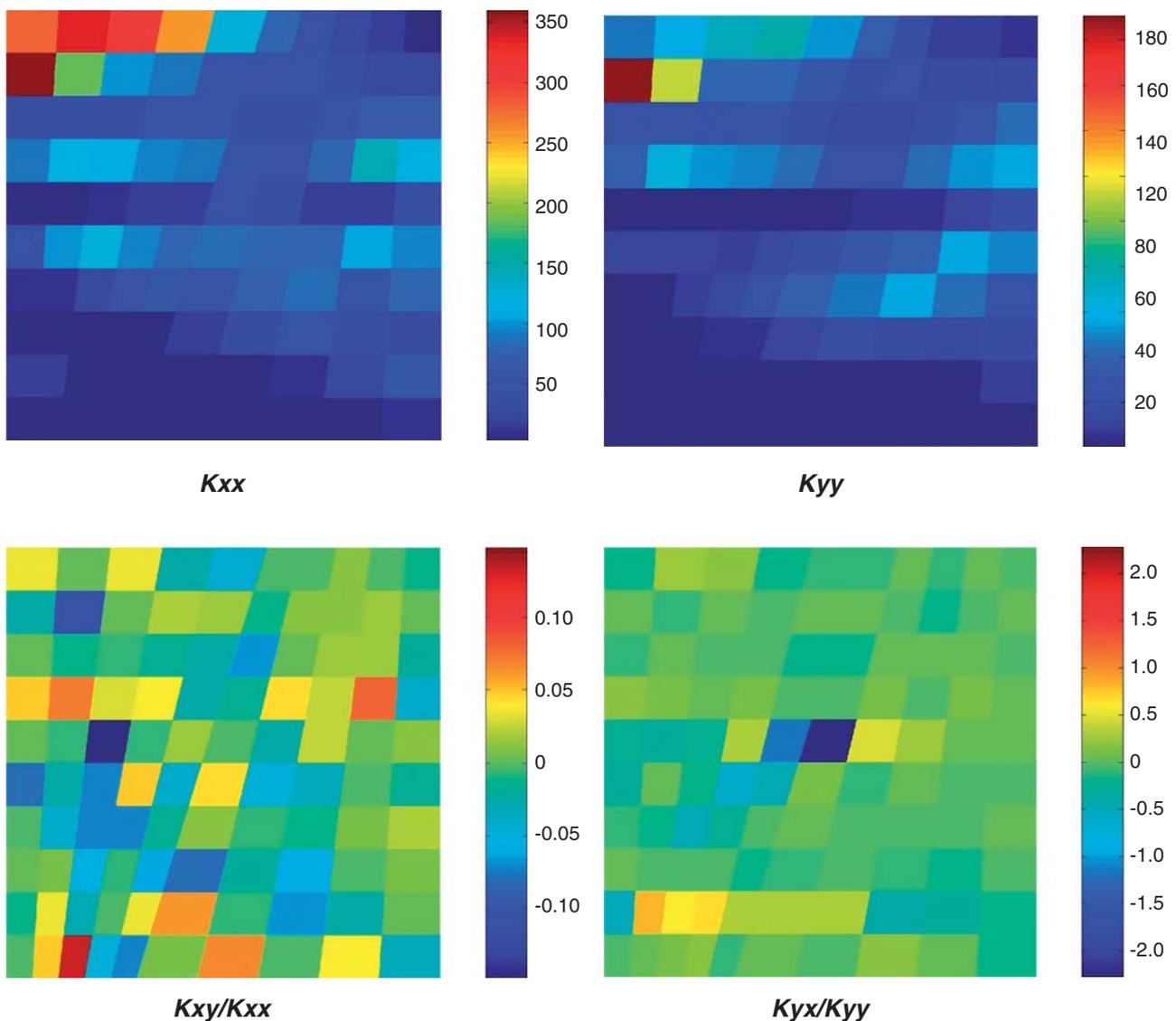


Figure 12

Cross-flow assessment during upscaling.

Figure 11 shows a  $100 \times 100$  areal geologic test section with a  $10 \times 10$  corner point simulation grid. The simulation grid is deliberately skewed compared to the natural axes of the geologic model. Figure 12 shows the cross-flow assessment during upscaling where, for instance,  $k_{xx}$  is compared to  $k_{yy}$ . The latter determines the strength of the coupling between the transverse pressure gradient and the flux in the  $x$ -direction. Similarly,  $k_{yy}$  and  $k_{xx}$  are contrasted. (Here,  $x$  and  $y$  denote the local  $i$  and  $k$  tangent directions on the coarse grid.) This is still work in progress, but we are exploring different means of using this information to help develop an unstructured

computational grid, similar to that of Figure 1, now driven by the transverse flux during an areal upscaling calculation.

## CONCLUSION

We have demonstrated through a series of case studies how local *a priori* error analyses can be used to guide a reservoir upgridding calculation. Various error calculations were utilized, as summarized in Table 1. Each provided insight to a different class of reservoir upgridding problem. We believe that the use of an error analysis, in conjunction with rule

based simulation grid design, is a powerful technique that complements our existing upscaling toolkit and can provide deeper integration between the reservoir engineering and geology domains.

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