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Dossier

Upscaling of Fluid Flow in Oil Reservoirs
Mise à l'échelle des écoulements dans des réservoirs pétroliers

Near-Well Upscaling for Reservoir Simulations

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Résumé — Mise à l'échelle au voisinage du puits pour simulations de réservoir — Les techniques de mise à l'échelle de perméabilité absolue (du modèle géologique ou géostatistique vers le modèle de simulation dynamique de réservoir) sont étudiées depuis longtemps dans la littérature. Il est généralement reconnu que la difficulté principale dans la mise à l'échelle est la dépendance des conditions aux limites, qui conduisent à des solutions non uniques. Ceci est particulièrement crucial au voisinage des puits, qui correspondent à des singularités de l'écoulement. Ainsi, à la suite d'une mauvaise mise à l'échelle de la perméabilité absolue aux abords du puits, les productivités de celui-ci, calculées par simulations en maillage fin et grossier, peuvent être très différentes.

Dans cet article, nous présentons des techniques de mise à l'échelle au voisinage du puits, qui permettent d'améliorer significativement la précision de simulation en maillage grossier pour la prédiction de la productivité du puits dans un milieu hétérogène. Cette technique a été développée pour puits complexes sur un maillage *corner-point geometry* (CPG) en 3D. En utilisant la mise à l'échelle au voisinage du puits, les résultats aux puits obtenus par simulations en maillage grossier et en maillage fin sont équivalents.

Abstract — Near-Well Upscaling for Reservoir Simulations — *Upscaling of absolute permeability from geological/geostatistical models to reservoir flow simulation models has been discussed for a long time. It has been generally recognized that the main difficulty in upscaling is the dependence of upscaling results on boundary conditions, which lead to a nonunique solution. As a consequence of inaccurate permeability upscaling, well performance prediction might be very different using fine and coarse grid simulations.*

In this paper, the technique of near-well upscaling, which can improve dramatically coarse grid simulation accuracy for well productivities in heterogeneous media, is presented. The proposed near-well upscaling has been developed for 3D complex wells on both Cartesian and corner-point geometry (CPG) gridblocks. Equivalent results on well productivity are obtained between coarse grid and fine grid simulations.

LIST OF SYMBOLS

CPG corner-point geometry
F flux
k permeability
p pressure
q well flow rate
T transmissibility
u pressure for a regular problem
V volume
WI well index
x, y, z coordinates
Γ_R reservoir boundary
Ω reservoir domain.

INTRODUCTION

Probabilistic techniques are widely used for modelling reservoir heterogeneities. However, the heterogeneities, such as permeability, generated on fine grids by geostatistical models, cannot be directly used for flow simulation, due to computational cost and memory storage capacity. Therefore, efficient techniques are needed to scale the fine grid petrophysical parameters up to the coarse grid. Among the upscaled parameters, absolute permeability is by far the most important property that affects flow behaviors. Upscaling of absolute permeability is the most often considered in the literature (Warren and Price, 1961; Begg *et al.*, 1989; King, 1989; Durlofsky, 1991; Nœtinger, 1994; Christie *et al.*, 2000; Holden *et al.*, 2000; Terpolilli and Hontans, 2000).

The main difficulty in permeability upscaling is the dependence of upscaling results on boundary conditions, which lead to a non unique solution. Consequently, well performance calculations might be very different using fine grid and coarse grid simulations. However, using the technique of near-well upscaling can improve significantly coarse grid simulation accuracy for well productivity prediction.

There are two reasons to make a special study for the upscaling in the near-well region:
– accuracy of well performance calculation depends directly on the accuracy of the near-well upscaling results;
– using near-well upscaling procedure, results are almost independent of boundary conditions, because well driven flow dominates in the well vicinity.

The impact of well configurations on upscaling results was discussed by several authors. White and Horne (1987) proposed a method to calculate wellblock transmissibilities. Their method was based on running several fine grid simulations with different boundary conditions. The pressures and fluxes from fine scale simulation were averaged to obtain pressures and fluxes on coarse gridblocks. However, as the numerical well index *(WI)* was not considered in that procedure, well performance calculations are inaccurate on coarse grid simulation. Palagi *et al.* (1993) used power law averaging for permeability to study the sensitivity of equivalent permeability to well configuration. But, as the near-well region is not specifically treated, their optimum power law coefficient, which is changed significantly from one configuration to another, cannot be determined.

The technique of near-well upscaling was first proposed by Ding (1995). A steady-state flow is simulated on fine gridblocks in the well vicinity, and the fine scale solution is then used to determine near-well block transmissibilities and numerical *WI* on the wellblocks. This technique was successfully used in field study (Durlofsky *et al.*, 2000). It was also applied to horizontal wells by Mascarenhas and Durlofsky (2000) and Muggeridge *et al.* (2002). Recently, the near-well upscaling has been developed on coarse corner-point geometry (CPG) gridblocks by Ding (2003).

The near-well upscaling is a necessary and efficient procedure to improve well productivity simulation with coarse grid model, comparing to fine scale results. Equivalent results of well performance are obtained between coarse and fine grid simulations, when near-well upscaling is used.

1 UPSCALING OF ABSOLUTE PERMEABILITY

Heterogeneities are generally generated by a geostatistical model in a fine grid with several to several hundred millions gridblocks. Such a grid cannot be directly used in dynamic flow simulation, due to high computational cost and memory storage. So, it is necessary to upscale to a large scale with coarse gridblocks for dynamic simulation.

Techniques for absolute permeability upscaling have been discussed for a long time. The first techniques consist in determining the intervals of equivalent permeability (Cardwell and Parson, 1945). Then, both algebraic methods (Journel *et al.*, 1986; Le Loc'h, 1987; Nœtinger, 1994; Romeu, 1994) and numerical approaches (Warren and Price, 1961; Begg *et al.*, 1989; Durlofsky, 1991; Gallouët and Guérillot, 1991) are developed. In algebraic methods, power-law averaging or a combination of power averaging in different directions is generally used. For the numerical approaches, we set up a single-phase flow simulation on fin gridblocks with specified boundary conditions and then determine what value of equivalent permeability yields the same flow rate as the fine grid calculation. The most commonly used boundary condition is the no-flow one (imposing constant pressures at inlet and outlet faces and no flow conditions on other faces) (Begg *et al.*, 1989). Periodic boundary condition (Durlofsky, 1991) is also used, especially when determining full tensor permeability. We can also use other boundary conditions for fine grid flow simulation (Gallouët and Guérillot, 1991). Other techniques that should be mentioned include dissipated energy (Bamberger, 1977), large scale averaging (Quintard and Whitaker, 1988), and renormalization method (King, 1989).

Among all these upscaling methods, the numerical ones are generally considered more accurate. However, the results of numerical upscaling depend greatly on boundary conditions used in fine grid simulation. Boundary effects in upscaling procedure have been greatly discussed in the literature (Christie *et al.*, 2000; Holden *et al.*, 2000; Terpolilli and Hontans, 2000). Optimum boundary condition is required in the numerical upscaling procedure.

On the other hand, it is generally recognized that upscaling of transmissibility is more accurate than upscaling of permeability for coarse scale flow simulation (Romeu and Nœtinger, 1995; Urgelli, 1998). According to Darcy's law, the equivalent transmissibility T_{ij} between two coarse gridblocks i and j can be determined by the quotient of equivalent flux F_{ij} and the difference in coarse grid pressure $p_j - p_i$:

$$T_{ij} = -\frac{F_{ij}}{p_j - p_i} \quad (1)$$

Once the equivalent flux and the equivalent coarse grid pressure are determined based on fine grid flow simulation, equivalent transmissibilities are obtained from Equation (1). Like permeability upscaling, the above equivalent transmissibility depends on flow scenarios (or boundary conditions). Using a particular boundary condition in upscaling procedure, for instance, no flow boundary condition (Begg *et al.*, 1989), does not usually correspond to the real flow scenario, especially in the near-well region. Other boundary conditions, such as periodic one (Durlofsky, 1991), do not correspond to the real scenario neither.

Accurate upscaling procedure should use the boundary condition as close as possible to reality. On a field scale, the flow pattern can be considered as two types: a "singular" flow pattern with high gradient pressure; and a "linear" flow pattern with low gradient pressure. Upscaling in "singular" flow region, which is usually the near-well region, is more important for well productivity calculation, while in most of the literature (Warren and Price, 1961; Begg *et al.*, 1989; King, 1989; Durlofsky, 1991; Holden *et al.*, 2000; Terpolilli *et al.*, 2000), the boundary conditions correspond to the "linear" flow pattern. So, it is important to consider "singular" flow behavior in the upscaling procedure for accurate well performance calculation.

2 NEAR-WELL UPSCALING PROCEDURE

2.1 Method of Near-Well Upscaling

The near-well "singular" flow behavior can be obtained by simulating a steady-state flow on fine gridblocks around a well. Using fine scale solution, the equivalent transmissibility obtained with Equation (1) adapts to the near-well flow pattern. For flow simulations, it is also necessary to introduce a numerical *WI* on the wellblock to relate the wellblock pressure, the wellbore pressure and the well flow rate. The *WI* on a coarse wellblock i can be determined by the following formula (Ding, 1996):

$$WI_i = \frac{q_i}{p_i - p_{wi}} \quad (2)$$

where q_i is the well flow rate on the wellblock i, p_i is the equivalent gridblock pressure, p_{wi} is the wellbore pressure on the wellblock i.

The near-well upscaling procedure is summarized as follows *(Fig. 1)*:

1 Choose a well (or several wells) to be investigated.
2 Select a near-well zone on fine gridblocks for flow simulation.
3 Simulate a steady-state flow in the selected zone with an outer boundary condition.
4 Select a near-well zone on coarse gridblocks for equivalent transmissibility calculation.
5 Calculate equivalent flux on coarse grid interface in the selected coarse grid zone.
6 Calculate equivalent coarse gridblock pressure in the selected coarse grid zone.
7 Calculate equivalent well flow rate and wellbore pressure on coarse wellblock.
8 Determination of near-well transmissibility values and numerical *WI* using Equations (1) and (2) with equivalent parameters obtained in steps 5, 6 and 7.

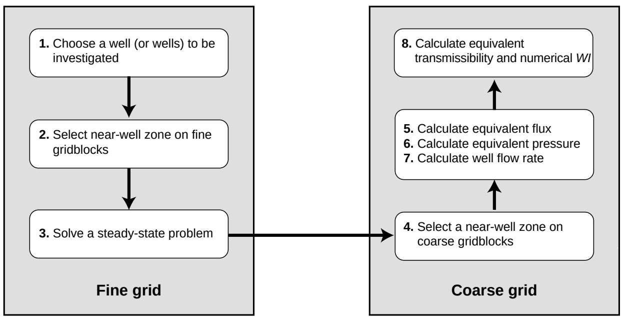

Figure 1

Scheme of near-well upscaling procedure.

The biconjugate gradient stabilized method is used for solving the linear system (Van Der Vorst, 1992). If both fine and coarse grids are Cartesian, calculations of equivalent parameters in steps 4, 5 and 6 are easy. Calculations of equivalent parameters on coarse CPG grid are presented in a later session.

2.2 Impacts of Boundary Conditions

Unlike other upscaling procedures, the results of near-well upscaling are almost independent of boundary conditions.

Without loss of generality, considering a 2D steady-state problem on a domain Ω, which contains a well, with two different Dirichlet boundary conditions:

$$\begin{cases} -\left(k_x \dfrac{\partial^2 p}{\partial x^2} + k_y \dfrac{\partial^2 p}{\partial y^2}\right) = q\delta(O) & (x,y) \in \Omega \\ p(x,y) = p_R(x,y) & (x,y) \in \Gamma_R \end{cases} \tag{3}$$

and:

$$\begin{cases} -\left(k_x \dfrac{\partial^2 \tilde{p}}{\partial x^2} + k_y \dfrac{\partial^2 \tilde{p}}{\partial y^2}\right) = \tilde{q}\delta(O) & (x,y) \in \Omega \\ \tilde{p}(x,y) = \tilde{p}_R(x,y) & (x,y) \in \Gamma_R \end{cases} \tag{4}$$

where k_x, k_y are permeabilities in x and y direction; the well is represented by a point sink/source and is located at the point O; q or $\tilde{q}$ is the well flow rate; $\Gamma_R = \partial\Omega$ is the boundary of Ω. The pressure behaviors of these two solutions are generally very close in the well vicinity. Let $u = p - C\tilde{p}$ with $C = q/\tilde{q}$, then u should be the solution of the following problem:

$$\begin{cases} -\left(k_x \dfrac{\partial^2 u}{\partial x^2} + k_y \dfrac{\partial^2 u}{\partial y^2}\right) = 0 & (x,y) \in \Omega \\ u(x,y) = p(x,y) - C\tilde{p}_R(x,y) & (x,y) \in \Gamma_R \end{cases} \tag{5}$$

This problem corresponds to steady-state flow on Ω without any well inside.

Well singularity is presented in Equations (3) and (4). The pressure and flux in the well vicinity are large and tend toward infinite values, when the distance to the well approaches zero. Equation (5), however, is a regular problem, because there is not any well singularity inside the domain Ω. The solution of Equation (5) and its derivatives are bounded, even in the near well region. Like solution splitting in a homogeneous media (Ding and Jeannin, 2001), the solution of Equation (3) can be split into a singular part $C\tilde{p}$ and a regular part u:

$$p = C\tilde{p} + u$$

In the well vicinity, the singular part usually dominates, and the contribution of the regular part can be neglected. This is particularly true for the calculation of pressure differences or pressure derivatives (fluxes). If the contribution of the regular solution u is neglected, equivalent transmissibilities determined by Equation (1) with boundary conditions in Equations (3) and (4) are almost identical, as:

$$\begin{aligned} T_{ij} &= -\frac{F_{ij}}{p_j - p_i} = -\frac{C\tilde{F}_{ij} + F_u}{C\tilde{p}_j - C\tilde{p}_i + u_j - u_i} \\ &\approx -\frac{C\tilde{F}_{ij}}{C\tilde{p}_j - C\tilde{p}_i} = -\frac{\tilde{F}_{ij}}{\tilde{p}_j - \tilde{p}_i} = \tilde{T}_{ij} \end{aligned} \tag{6}$$

where F_{ij}, ($\tilde{F}_{ij}$) is the flux across the interface Γ_{ij} for Equation (3) (Equation (4)); F_u is the flux across the interface Γ_{ij} corresponding to the singular solution Equation (5); p_i, p_j ($\tilde{p}_i$, $\tilde{p}_j$) are equivalent coarse gridblock pressures of Equation (3) *(Eq. (4))*; and u_i, u_j are the equivalent coarse gridblock pressures of the Equation (5).

The impacts of boundary conditions are very small on transmissibility calculation in near-well upscaling procedure. This conclusion was also checked by numerical tests (Ding, 1995). In practise, either transmissibilities T_{ij} or $\tilde{T}_{ij}$ can be used.

2.3 An Example

Considering a 2D problem, the reservoir permeability is generated using a geostatistical model on 99×99 fine gridblocks with short correlation length *(Fig. 2)*. Reservoir and fluid data are given in Table 1. Fourteen vertical wells are considered in this field with five injectors (noted by INJ1-INJ5) and nine producers (noted by PROD1-PROD9). Bottom hole well flowing pressures are imposed on all wells to calculate well flow rates. Results of the fine grid simulation are considered as the reference solution. A 33×33 coarse grid is constructed based on this fine grid, and each coarse gridblock contains nine fine blocks (three blocks in the x-direction and three blocks in the y-direction). Both fine and coarse grids are Cartesian. Two upscaling procedures are compared: “linear flow” upscaling and near-well upscaling. The “linear flow” upscaling procedure uses no flow boundary conditions on the edges and constant pressure at the inlet and outlet faces (Begg *et al.*, 1989). For coarse grid simulation with the near-well upscaling procedure, equivalent transmissibilities in far well region are calculated using the “linear flow” procedure.

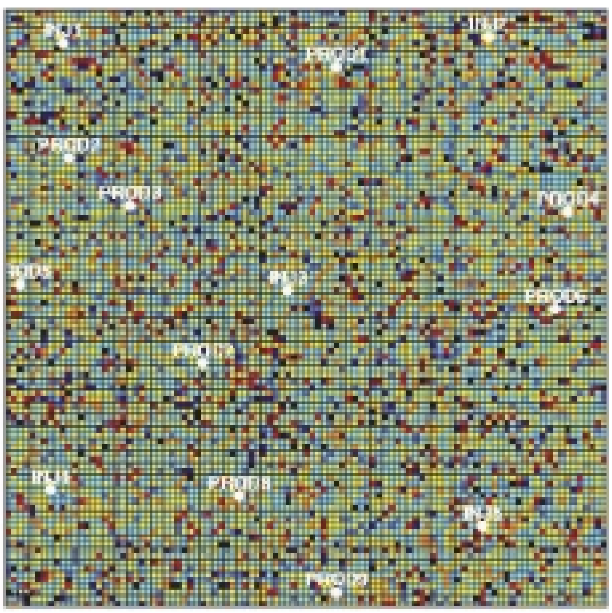

Figure 2

Heterogeneity on 99×99 fine gridblocks.

TABLE 1

Data for the simulation

Field permeability	Lognormal distribution (mean = 5; variance = 1) Correlation length = 10 m
Reservoir geometry	Fine gridblock size: $dx = dy = 10$ m Coarse gridblock size: $dx = dy = 30$ m Height = 5 m
Formation properties	Initial pressure = 1.05E+7 Pa Porosity = 0.2
Well conditions	Injection well pressure = 1.15E+7 Pa Production well pressure = 9E+6 Pa
Fluid properties	Oil viscosity = 0.001 Pa·s Water viscosity = 0.001 Pa·s Oil density = 906 kg/m^3 Water density = 1000 kg/m^3

2.3.1 Single-Phase Flow Simulation

Single-phase flow is first tested. Figure 3 compares well flow rate on production wells given by three simulations: the fine grid simulation and two coarse grid simulations using "linear flow" upscaling procedure and near-well upscaling procedure. Errors caused by the "linear flow" upscaling procedure reach about 30% on average, while errors obtained with the near-well upscaling procedure are only 1-2%. The improvement by using near-well upscaling is significant. Errors of "linear flow" upscaling are reduced by a factor of ten.

For certain wells, for instance, the production well PROD1, the coarse grid simulation result obtained with the "linear flow" upscaling procedure is far from the reference solution. This large error results mainly from two reasons:

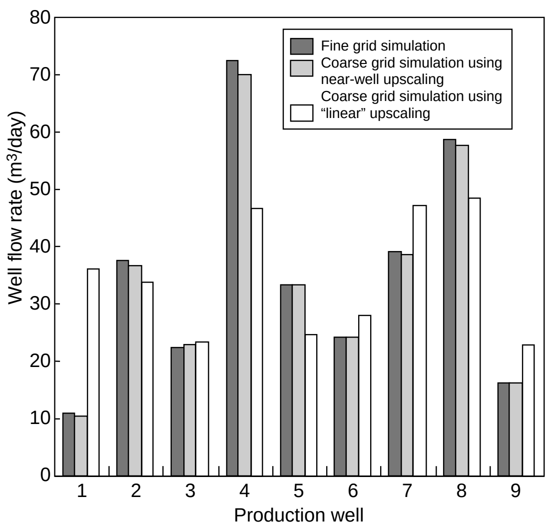

Figure 3

Comparison on single-phase flow simulation.

– well PROD1 is not centred in the coarse gridblock;
– and the contrast is strong between permeabilities on the fine grid wellblock and its neighbours *(Fig. 4)*.

Nevertheless, the near-well upscaling can take into account the well off-centred in a gridblock, and the relation between the fine grid and coarse grid wellblock permeabilities can be conveniently handled through the near-well upscaling procedure.

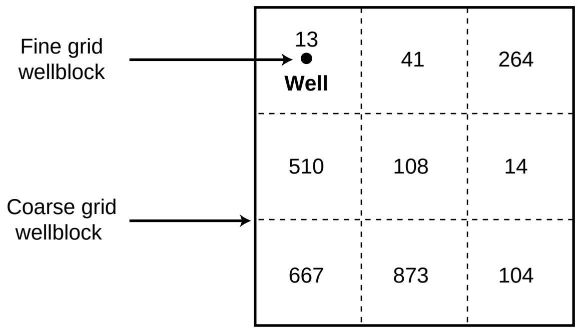

Figure 4

Permeability values around the well PROD1.

2.3.2 Two-Phase Flow Simulation

Water is injected in the oil reservoir. Relative permeability and capillary pressure are given in Figure 5. Figure 6 shows the water cut for the nine production wells. Again, the simulation using near-well upscaling procedure approaches the fine grid results much better. The water breakthrough time can be calculated with satisfactory accuracy with the near-well upscaling procedure.

Coarse grid simulations in multi-phase flow can also be improved using near-well upscaling procedure due to the two following reasons:

– the equivalent permeability between two wells, which is related to breakthrough time, is conveniently handled;
– the distance between two wells is taken into account in the near-well upscaling procedure through consideration of modelling of off-centred well.

2.4 Accurate Fine Grid Simulation

Using the near-well upscaling procedure, equivalent results are obtained between coarse and fine scale simulations. The accuracy of well performance calculation using coarse grid simulation depends directly on the accuracy of fine grid simulation. But how can one model accurately a well on fine gridblocks in heterogeneous media?

One solution is using grid refinement on the fine gridblocks associated with the near-well upscaling procedure (Ding, 1995). When the fine gridblocks are refined around the well, permeability variation is relatively homogeneous

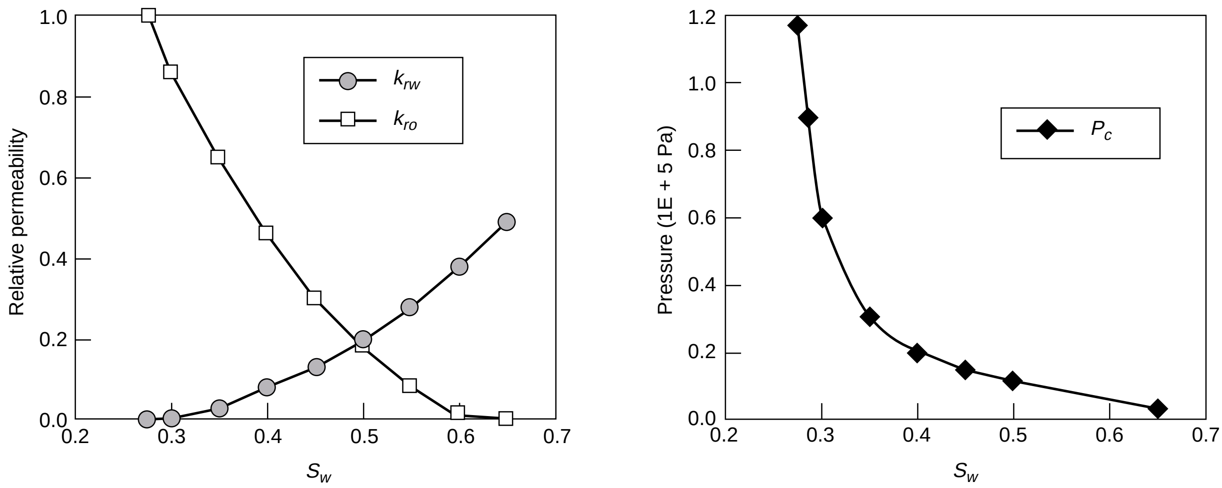

Figure 5

Relative permeability and capillary pressure.

from block to block. The well model developed for homogeneous media (Peaceman, 1983; Ding, 1996) can be used and considered accurate on the refined gridblocks. Then, we apply the near-well upscaling from the refined grid to the fine gridblocks. Like upscaling between coarse and fine grids, results on the well are equivalent between the fine gridblock and its refinement. Therefore, a suitable well model is obtained for flow simulation on fine gridblocks.

2.5 Upscaling Around Complex Wells

There is no restriction for application of the proposed near-well upscaling procedure to complex wells. In fact, accuracy of the absolute permeability upscaling around a complex well depends on accuracy of the fine grid simulation. The technique of simulation of complex wells using a reservoir simulator (on a fine grid) is described in Ding (1996). When necessary, grid refinement can be used to improve well simulation accuracy as discussed in the above session.

3 NEAR-WELL UPSCALING ON 3D CPG GRIDBLOCKS

The fine grid, issued from a geostatistical model to define heterogeneity, is generally Cartesian, but the coarse grid used for flow simulation is usually corner-point geometry. Upscaling on CPG gridblocks in 3D is more difficult because of grid distortion and lack of superposition between coarse gridblocks and fine gridblocks.

3.1 Numerical Schemes

Transmissibility upscaling is closely related to the numerical scheme on coarse scale simulation. Two-point flux approximation schemes are usually considered as inadequate for flow modelling on distorted gridblocks such as CPG. Multi-point flux approximation schemes are therefore used (Aavastmark *et al.*, 1998; Edwards and Rogers, 1994). However, these schemes need more CPU time and memory storage capacity.

In the well vicinity, a convenient numerical scheme should adapt to model “singular” flow behavior, which has a stronger impact on well performance accuracy than grid distortion. An analysis of numerical schemes for near-well flow modeling was presented by Ding and Jeannin (2001). Although the two-points scheme is inaccurate for flow modelling on CPG gridblocks, it can still be considered suitable in the well vicinity if it can handle correctly “singular” flow behaviour. Equivalent transmissibility upscaled by Equation (1) adapts to two-point schemes for near-well flow simulation.

3.2 Calculation of Equivalent Parameters

3.2.1 Calculation of Equivalent Flux

In most cases, generations of both fine gridblocks and coarse gridblocks are constrained by the same stratigraphical layers in the vertical direction. Both coarse grids and fine grids discretize the same geological layers. So, each coarse gridblock groups a series of fine gridblocks in the vertical direction *(Fig. 7)*. The topological approach can assure the coherence between fine grid and coarse grid simulations. With the topological constraint, it is not necessary to calculate 3D geometrical intersections between fine and coarse gridblocks.

Using topological constraint, flow on coarse gridblocks in x or y directions is limited in each layer. A coarse gridblock

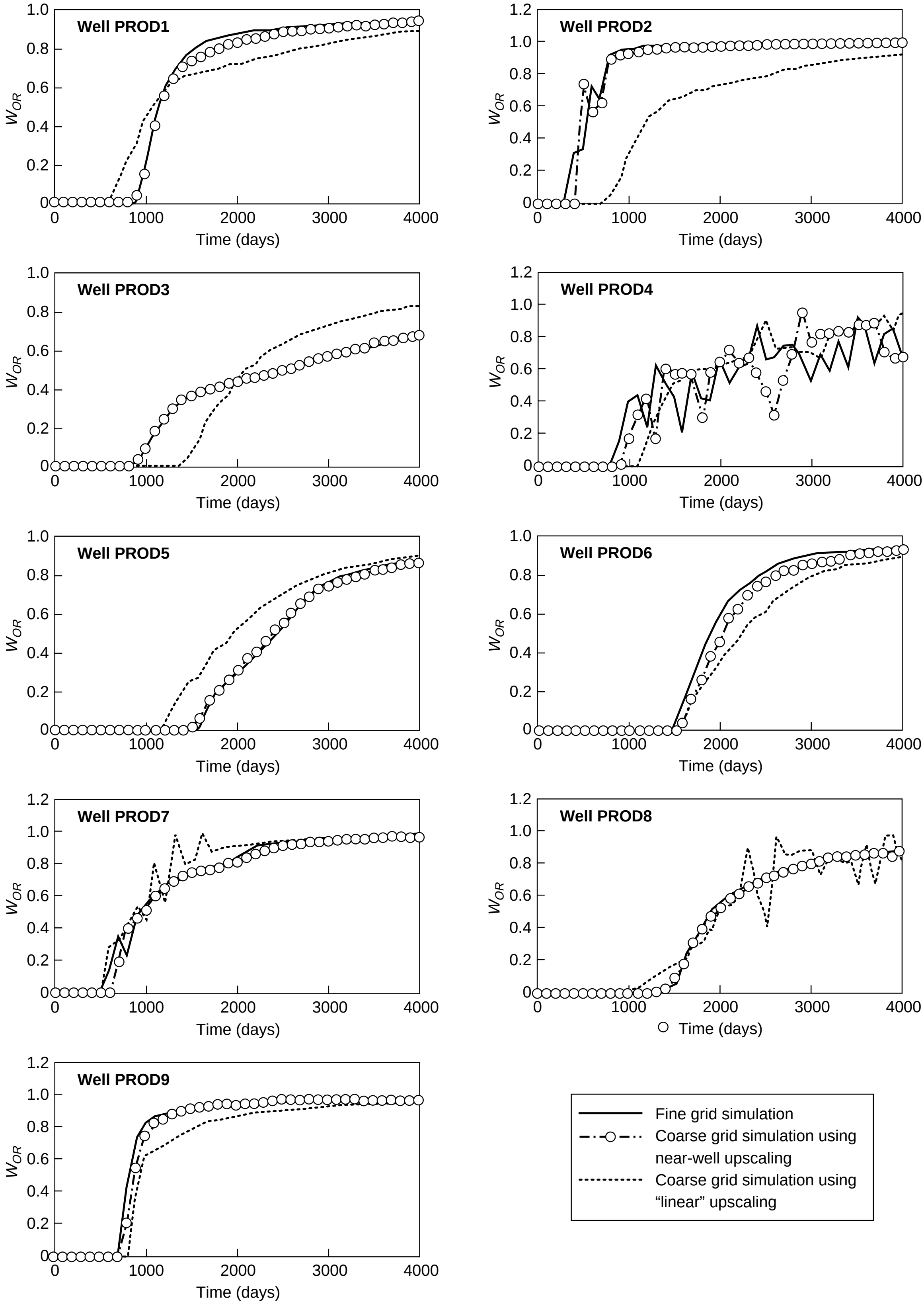

Figure 6

Water-oil ratio on production wells for two-phase flow simulation.

communicates only with coarse gridblocks in the same layer. The problem becomes a 2D one.

To calculate equivalent flux in the z direction, the coarse gridblock interface is projected onto the horizontal *xy* plane. Topological constraint is used to determine the vertical position of the coarse block interface, which coincides with fine gridblock interfaces and is represented by a series of horizontal facets *(Fig. 8)*. So, the position of a coarse block interface in the vertical direction can be easily determined.

In cases where topological constraint is not respected, equivalent flux can be calculated by determining geometric intersection between gridblocks or using numerical approach for integral calculation (Ding, 2003). In all methods, field velocity should be determined based on fine grid simulation.

3.2.2 Calculation of Equivalent Pressure

Upscaling procedure is usually based on a steady-state flow simulation, and the variation of porosity is neglected. Therefore, the equivalent coarse gridblock pressure can be calculated by a volumetric average as follows:

$$\bar{p} = \frac{1}{V} \int_V p(x,y,z) \mathrm{d}V \qquad (7)$$

where $\bar{p}$ is the equivalent pressure on the coarse gridblock, V is the volume of this block.

To calculate the integral on a volume V using a numerical method, the geometry of the coarse gridblock is first transformed to a standard cube. The Gauss-Legendre formula is then applied on the standard cube for the numerical integral calculation.

3.2.3 Calculation of Equivalent Well Parameters

Well flow rate is calculated on fine gridblocks using fine scale simulation. However, these cannot be used directly to calculate well flow rate on coarse gridblocks. Depending on well configuration such as well length or wellbore radius, flow rate distribution along the well can be determined from fine grid simulation. Once intersections between the well trajectory and coarse gridblocks are determined, well flow rate on a coarse gridblock can be obtained, because flow rate distribution along the well length is known.

Infinite well conductivity condition is generally used in fine grid simulation. In this case, the wellbore pressure is always a constant.

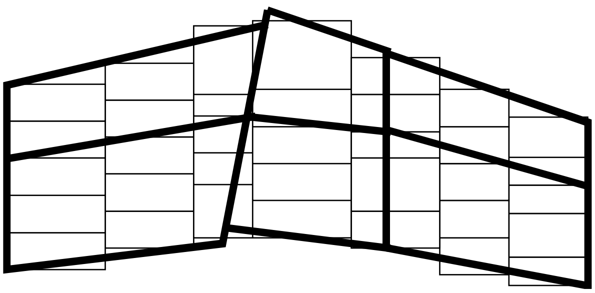

Figure 7

Topological constraint in vertical direction. The 1st layer in the coarse grid regroups the first two layers in the fine grid. The 2nd layer in the coarse grid regroups the last three layers in the fine grid.

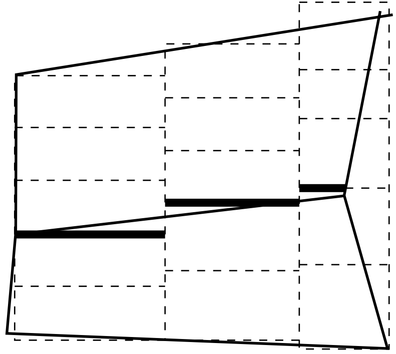

Figure 8

Flux upscaling in vertical direction under topological approach.

CONCLUSIONS

Techniques for upscaling of absolute permeability from fine grid issued from geostatistical model to coarse grid for flow simulation are discussed. On field scale, two types of flow pattern should be considered in upscaling procedures: “linear” flow pattern with low pressure gradient in far-well region and “singular” flow patterns with high pressure gradients in near-well regions. Upscaling with “singular” flow patterns, which relates directly to well performance calculation, plays an important role in coarse scale flow simulation.

The technique of near-well upscaling, which uses “singular” flow patterns in the upscaling procedure, can improve significantly coarse grid simulation accuracy. In the near-well upscaling procedure, equivalent transmissibilities in addition to numerical *WI* on the wellblock are calculated using a fine grid simulation in the vicinity of the well. Impacts of boundary conditions on the upscaling results are small. A two-point flux approximation scheme can be applied for flow simulation in the vicinity of the well on corner-point geometry, and near-well transmissiblity upscaling for the two-point scheme is presented. This procedure has been used for modelling of 3D complex wells on both Cartesian and corner-point geometry coarse gridblocks. Various approaches have been developed to overcome the difficulty of arbitrary intersections between fine and coarse gridblocks in the upscaling procedure. The near-well upscaling concept can also be used for the well modelling on fine gridblocks in heterogeneous media to improve fine grid simulation accuracy.

The importance and the necessity of near-well upscaling for coarse grid flow simulation are shown. Without the near-well upscaling procedure, errors on coarse grid simulation are generally high, while using the near-well upscaling procedure, errors can be reduced by a factor of ten. The near-well upscaling procedure can assure the coherence of well performance calculation between coarse and fine grid simulations. Upscaling of absolute permeability in the near-well region improves also coarse grid simulation accuracy in multi-phase flow.

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