

# OPTIMIZATION OF A BRAYTON-JOULE ENGINE SUBJECT TO MASS TRANSFER LIMITATIONS DUE TO PRESSURE LOSSES

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OPTIMISATION D'UN MOTEUR BRAYTON-JOULE  
SOU MIS À DES LIMITATIONS DE TRANSFERT  
DE MASSE PAR SUITE DE PERTES DE PRESSION

Dans la publication proposée ci-après, nous étudions un éventuel moteur solaire. L'influence du transfert de matière couplé aux pertes de pression sur le maximum de puissance pouvant être obtenu par un tel système est analysée.

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In the paper proposed, here we focus on a possible solar engine. We study the influence of mass transfer coupled to pressure losses on the maximum available power of the system.

OPTIMIZACIÓN DE UN MOTOR BRAYTON-JOULE  
SOMETIDO A LIMITACIONES DE TRANSFERENCIA  
DE MASA COMO CONSECUENCIA DE PÉRDIDAS  
DE PRESIÓN

En el presente artículo se estudia un posible motor solar. Se analiza la influencia de la pérdida de materia conjuntamente con las pérdidas sobre la potencia máxima que se puede obtener mediante semejante sistema.

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## INTRODUCTION

Optimal use of energy is prospected for the near future range, by using more efficient gas turbine system; for example, this is an industrial question now open, as can be seen, in a recent paper. Bannister *et al.* [3] suggest to reduce use of cooling air, and to augment component efficiencies of gas turbine system. Valenti [4] also, suggests to develop more efficient and environmentally clean system, by using higher temperatures in order to control burned emitted gases.

In a preceding communication, we focus on the possible use of solar energy [5]. The same study has been completed, by exploring the sensitivity of maximum power produced by the system to heat regeneration process [6], or to influence of the finite size of the heat reservoirs [1].

In the model proposed here, we focus again on a possible solar engine (Fig. 1); this is quite similar to what has been presented at the ENSEC conference Cracow, Poland, but we study the influence of mass transfer coupled to pressure losses, on the maximum obtainable power of the system. Representation of the corresponding cycle is given in Figure 2, in a temperature —entropy diagram; this corresponds to some proposed matter of Radcenco [2], that was only focused on pressure losses associated to compressor and turbine.

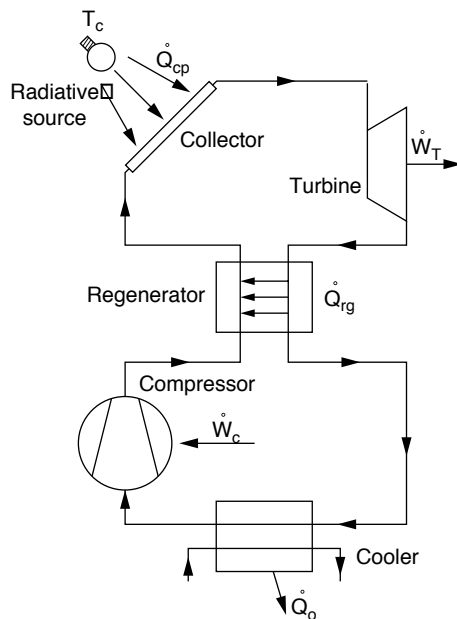


Figure 1

Schematic representation of the radiative engine.

## NOMENCLATURE

$p$	pressure [Pa]
$T$	t-temperature [K], [°C]
$\dot{Q}$	thermal energy flux [W]
$\dot{W}$	mechanical power [W]
$\dot{S}$	entropy flux [W/K]
$c$	specific heat [J/(kgK)]
$\gamma$	isentropic coefficient
$\dot{m}$	mass flux
$k$	heat transfer coefficient [W/(Km <sup>2</sup> )]
$A$	surface [m <sup>2</sup> ]
$K$	ratio [-]
$\eta$	efficiency
$NUT$	number of thermal unit transfer
$\beta$	pressure ratio in cycle (compressor and turbine)
$\psi$	relative pressure drop [-]
$\Pi$	exergetic loss [-].

### Subscript and superscript

<i>rad</i>	radiative
<i>opt</i>	optimum
<i>min</i>	minimum
<i>m</i>	mean
<i>c</i>	Carnot
<i>ex</i>	exergetic
<i>h</i>	pneumatic
<i>T</i>	turbine

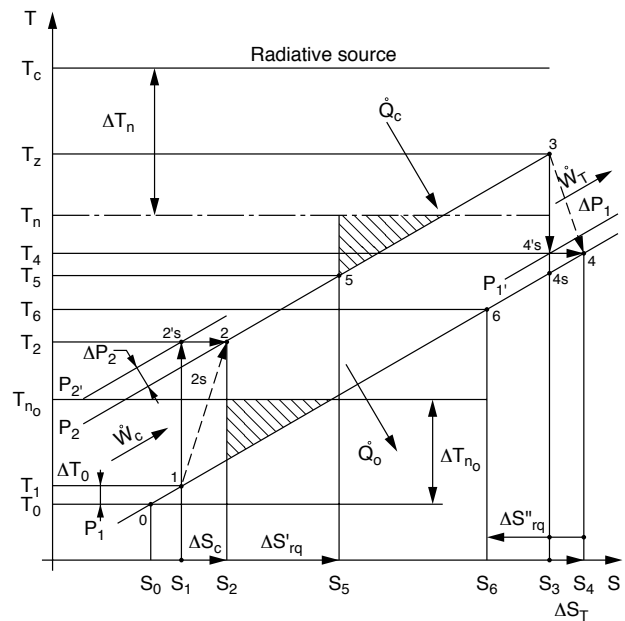


Figure 2

Cycle of the radiative engine representation in  $T$ - $s$  diagram.

<i>C</i>	compressor
<i>S</i>	system (sun)
<i>cp</i>	collector
<i>0</i>	cooler
<i>rg</i>	regenerator
<i>t</i>	thermic.

## 1 PROPOSED MODEL

Using a thermodynamical approach based on previous study [2], applied to stationary state of the engine, various energy fluxes can be expressed:

$$\dot{Q}_{cp} = k_{rad} \cdot A_{cp} \cdot T_C \cdot \left[ 1 - \left( \frac{T_m}{T_C} \right)^4 \right] = \dot{m} \cdot c_p \cdot (T_3 - T_5)$$

with

$$k_{rad} = \varepsilon_{rad} \cdot \sigma_0 \cdot T_C^3$$

$$T_m = (T_3 - T_5) / \ln(T_3/T_5)$$

$$\dot{Q}_0 = k_0 \cdot A_0 \cdot T_0 \cdot \left( \frac{T_{m0}}{T_0} - 1 \right) = \dot{m} \cdot c_p \cdot (T_6 - T_1)$$

with

$$T_{m\ 4-6} = (T_4 - T_6) / \ln(T_4/T_6)$$

$$T_{m\ 2-5} = (T_2 - T_5) / \ln(T_2/T_5)$$

It is to be noticed that the mass flux conservation imposes:

$$\dot{m} = K_C \cdot \Delta p_2 = K_T \cdot \Delta p_1$$

If  $K_C$ ,  $K_T$ , and  $P_1$  are given parameters, and expressing the pressure losses on an adimensional form:

$$\psi_{ij} = \frac{\Delta p_i}{p_j}$$

It comes:

$$\dot{m} = K' \cdot \psi_{21} = K'' \cdot \psi_{11}$$

with:

$$K' = K_C \cdot p_1, \quad K'' = K_T \cdot p_1, \quad \bar{K}_h = \frac{K''}{K'}$$

so

$$\psi_{21} = \bar{K}_h \cdot \psi_{11}$$

Some cycle temperatures, can be easily expressed in terms of parameters ( $T_0$ ,  $\Delta T_0^{min}$ ,  $\tau$ ,  $\beta$ ) and variables ( $T_3$ ,  $T_5$ ,  $\psi_{11}$ , or  $\psi_{21}$ ) due to following relations:

$$T_1 = T_0 + \Delta T_0^{min}$$

$$T_2 = T_1 \cdot \tau \cdot \left( 1 + \frac{\psi_{21}}{\beta} \right)^{\frac{\gamma-1}{\gamma}}$$

$$T_4 = T_3 \cdot (1 + \psi_{11})^{\frac{\gamma-1}{\gamma}}$$

$$T_6 = T_4 - T_5 + T_2$$

In addition, the preceding variables are related through two non linear equations obtained by expressing the two report of  $NUT_0$ , to  $NUT_{cp}$  (respectively  $NUT_{rg}$ ):

$$\frac{NUT_0}{NUT_{cp}} = \bar{K}_{tcp} = \bar{k}_{cp} \cdot \bar{A}_{cp} = \frac{k_0 \cdot A_0}{k_{rad} \cdot A_{cp}}$$

$$= \frac{T_C}{T_0} \cdot \frac{1 - (T_m/T_C)^4}{T_{m0}/T_0 - 1} \cdot \frac{T_6 - T_1}{T_3 - T_5}$$

$$\frac{NUT_0}{NUT_{rg}} = \bar{K}_{rg} = \bar{k}_{rg} \cdot \bar{A}_{rg} = \frac{k_0 \cdot A_0}{k_{rg} \cdot A_{rg}}$$

$$= \frac{T_{m4-6} - T_{m2-5}}{T_{m0} - T_0} \cdot \frac{T_5 - T_2}{T_6 - T_1}$$

By the same way entropy analysis has been performed. It results the following entropy fluxes generated in each successive elements of the engine:

– the entropy flux in the compressor:

$$\dot{S}_c = c_p \cdot \frac{\gamma-1}{\gamma} \cdot K' \cdot \psi_{21} \cdot \ln \left( 1 + \frac{\psi_{21}}{\beta} \right)$$

– the entropy flux in the turbine:

$$\dot{S}_T = c_p \cdot \frac{\gamma-1}{\gamma} \cdot K' \cdot \psi_{21} \cdot \ln(1 + \psi_{11})$$

– the entropy flux in the regenerator:

$$\dot{S}_{rg} = c_p \cdot K' \cdot \psi_{21} \cdot \ln \frac{T_6 \cdot T_5}{T_4 \cdot T_2}$$

- the entropy flux exchanged at the hot side of the engine:

$$\dot{S}_{\Delta T_m} = c_p \cdot K' \cdot \psi_{21} \cdot \frac{(T_3 - T_5) \cdot (T_C - T_m)}{T_m \cdot T_C}$$

- the entropy flux exchanged at the cold side of the engine:

$$\dot{S}_{\Delta T_{m0}} = c_p \cdot K' \cdot \psi_{21} \cdot \frac{(T_6 - T_1) \cdot (T_{m0} - T_0)}{T_{m0} \cdot T_0}$$

If  $T_0$ , is the reference (ambient) temperature, entropy analysis can be completed, by exergy analysis, noting that in general:

$$\dot{\Pi}_i = T_0 \cdot \dot{S}_i$$

Non dimensionnal form can be expressed relative to the dispoible energy incoming on  $\dot{Q}_S$ :

$$\eta_S = \frac{\dot{W}}{\dot{Q}_S} = \eta_{ex} \eta_{cp} \eta_c$$

with

$$\dot{W} = \dot{Q}_{cp} - \dot{Q}_0$$

$$\eta_{cp} = \frac{\dot{Q}_{cp}}{\dot{Q}_S}$$

$$\eta_c = 1 - \frac{T_0}{T_m}$$

Numerical calculation has been done using Matlab, results are traited on files by Excell. Relative precision of the calculation is better than 1%.

All the intermediate variables (for example  $T_2$ ,  $T_4$ ,  $T_5$ ,  $T_6$ ) and fluxes are calculated for a given set of parameters:

- the pressure ratio  $\beta$  has values in the range (3, 10);
- the radiative temperature of the source can be varied in the range (1500 K; 6000 K);
- the  $NUT$  ratio  $\bar{K}_{tc}$ ,  $\bar{K}_{tr}$ , can be varied in the range (0,1; 10);
- the mass conductance ratio  $k = \frac{K_T}{K_c}$  is varied in the range (0.5, 2).

## 2 RESULT OF CALCULATION

### 2.1 Influence of $\Psi$ for a given $\beta$

The sensitivity analysis developped shows us that,  $\Psi$  is an impotant parameter that represents the relative

pressure losses in the system ( $\Psi \equiv \Psi_{11}$ ). All other parameters remaining constants, it exists a maximum of mechanical power; this optimum is an increasing function of the  $\beta$  parameter (Fig. 3).

Generally for the actual existing engine the  $\psi$  values are in the range 0.12-0.24; the corresponding  $\beta$  are of the order of this value at maximum. It seems possible when  $\beta$  is greater than four, to augment the  $\psi$  value in the corresponding system, to obtain the maximum power.

Figures 4, 5, 6 gives the corresponding values of the hot flux at the collector, flux at the cold side, and global efficiency of the engine. The two first are increasing functions of  $\psi$ , the last one is decreasing.

In the presented calculation the turbine and compressor efficiencies are intermediate variables (Figs. 7 and 8). The calculation results show that they decrease with  $\psi$ , but increase with  $\beta$ . The turbine efficiency is more affected by  $\psi$  than the compressor efficiency.

The results of exergy analysis are presented on figures 9 to 13. Figures 9 and 10 show that exergy loss in the turbine and compressor is increasing function of  $\psi$ , but decreasing function of  $\beta$ . More exergy is destroyed in the turbine. Figures 11 and 12 represent exergy losses at the collector, respectively the cold side. More exergy is destroyed at the collector, but  $\psi$  is not influencing (only  $\beta$ ); at the cold side,  $\beta$  has a poor influence, and the exergy loss increases with  $\psi$ . In the regenerator heat exchanger exergy loss is negligible. Figure 13 summarizes the preceding results, by presenting the exergetic efficiency, strongly decreasing with  $\psi$ , and increasing with  $\beta$ .

### 2.2 Influence of $\bar{K}_{tc}$

We observe on Figure 14, than greater value of  $\bar{K}_{tc}$  is favourable in term of power until a  $\beta$  value near six; after what, there is approximately the same increasing tendency for both values of  $\bar{K}_{tc}$ . But the  $\psi$  corresponding to the optimum differs strongly, particularly at low value of  $\beta$ , where small value of  $\bar{K}_{tc}$  is in favour of  $\psi$  optimum actually used. We notice that at high value of  $\beta$ , the conclusion is reversed, and the  $\psi$  values are far from the actually used values. Figure 15 shows the corresponding behaviour of the global efficiency of the system.

### 2.3 Influence of $\Delta T_0$

Figure 16 illustrates that  $\Delta T_0$  influence is not determinant on the maximum power. We note a small

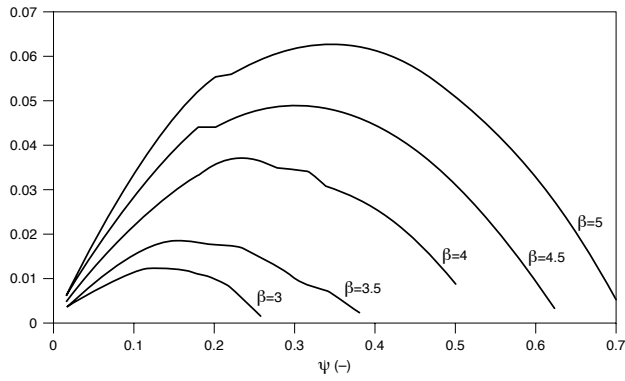


Figure 3  
 Engine Power  $T_c = 1500\text{ K}$ ,  $k = 1$ ,  $T_0 = 300\text{ K}$ ,  $K_{tc} = 0.01$ ,  $K_{tr} = 0.5$ .

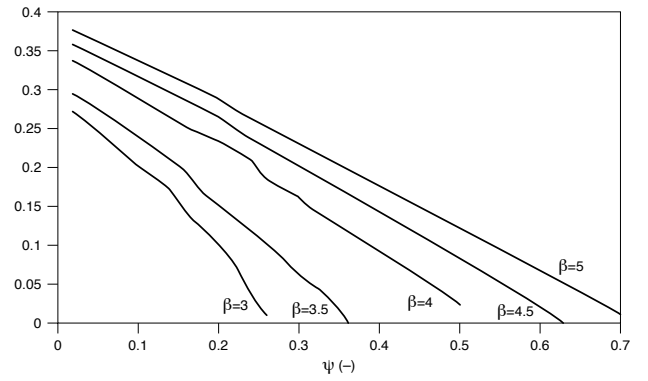


Figure 6  
 Global efficiency of the cycle  $T_c = 1500\text{ K}$ ,  $k = 1$ ,  $T_0 = 300\text{ K}$ ,  $K_{tc} = 0.01$ ,  $K_{tr} = 0.5$ .

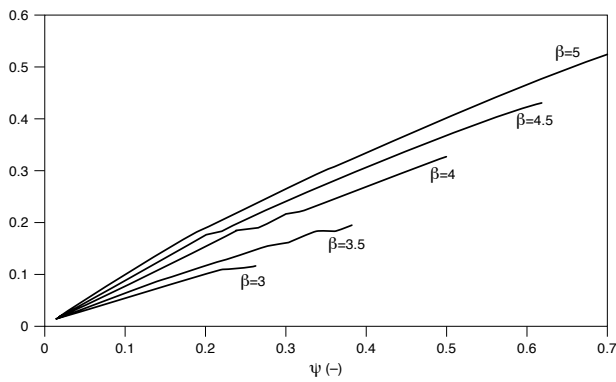


Figure 4  
 Heat flux at the collector  $T_c = 1500\text{ K}$ ,  $k = 1$ ,  $T_0 = 300\text{ K}$ ,  $K_{tc} = 0.01$ ,  $K_{tr} = 0.5$ .

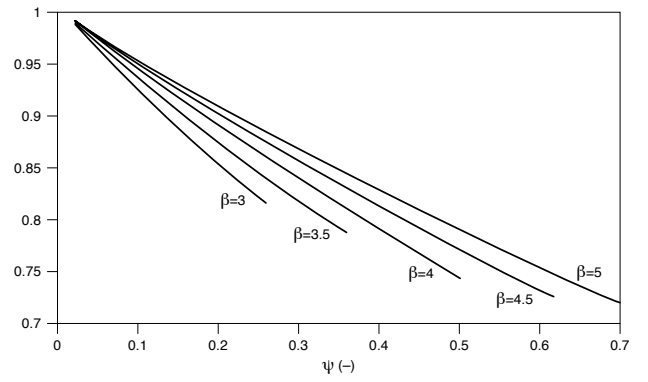


Figure 7  
 Turbine efficiency  $T_c = 1500\text{ K}$ ,  $k = 1$ ,  $T_0 = 300\text{ K}$ ,  $K_{tc} = 0.01$ ,  $K_{tr} = 0.5$ .

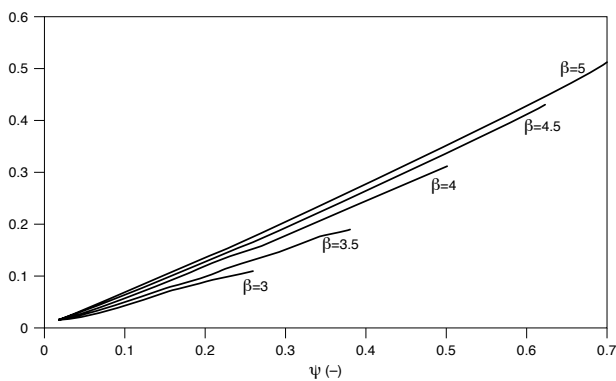


Figure 5  
 Heat flux at the cold side  $T_c = 1500\text{ K}$ ,  $k = 1$ ,  $T_0 = 300\text{ K}$ ,  $K_{tc} = 0.01$ ,  $K_{tr} = 0.5$ .

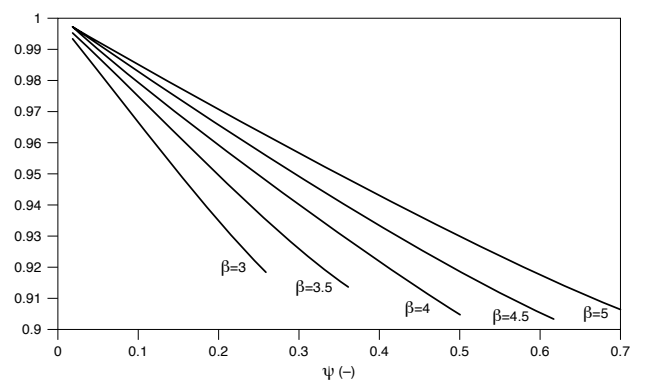


Figure 8  
 Compressor efficiency  $T_c = 1500\text{ K}$ ,  $k = 1$ ,  $T_0 = 300\text{ K}$ ,  $K_{tc} = 0.01$ ,  $K_{tr} = 0.5$ .

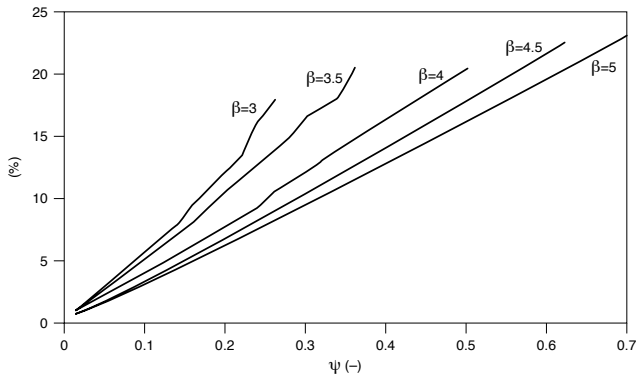


Figure 9  
Turbine exergy loss  $T_c = 1500 K$ ,  $k = 1$ ,  $T_0 = 300 K$ ,  $K_{tc} = 0.01$ ,  $K_{tr} = 0.5$ .

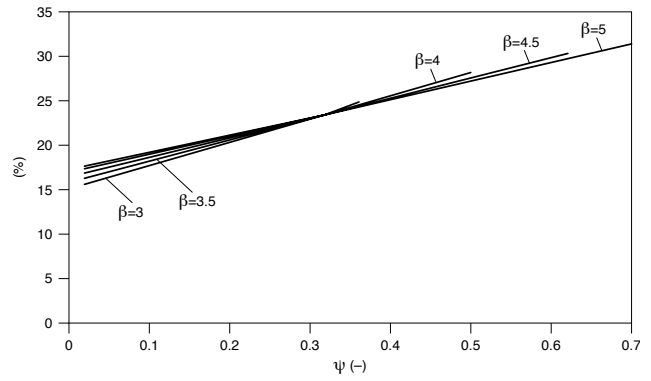


Figure 12  
Cold side exergy  $T_c = 1500 K$ ,  $k = 1$ ,  $T_0 = 300 K$ ,  $K_{tc} = 0.01$ ,  $K_{tr} = 0.5$ .

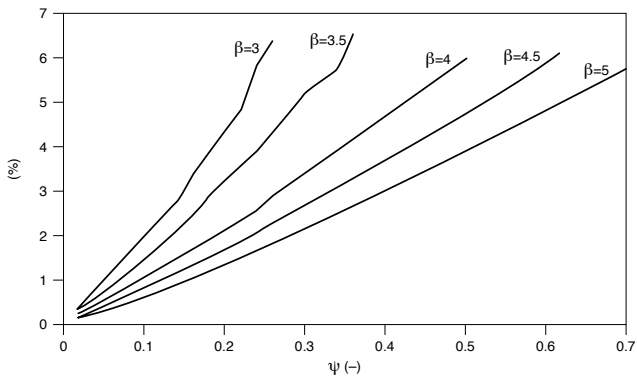


Figure 10  
Compressor exergy loss  $T_c = 1500 K$ ,  $k = 1$ ,  $T_0 = 300 K$ ,  $K_{tc} = 0.01$ ,  $K_{tr} = 0.5$ .

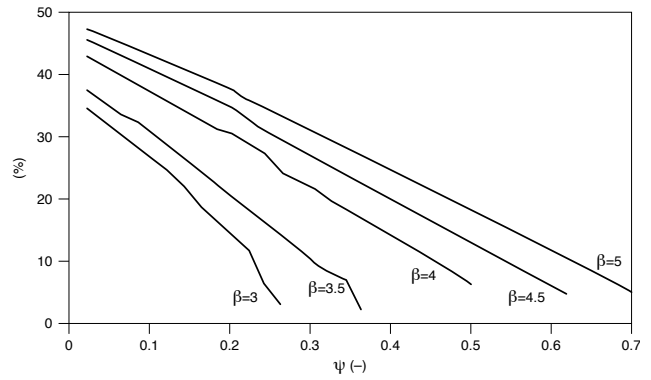


Figure 13  
Global exergetic efficiency  $T_c = 1500 K$ ,  $k = 1$ ,  $T_0 = 300 K$ ,  $K_{tc} = 0.01$ ,  $K_{tr} = 0.5$ .

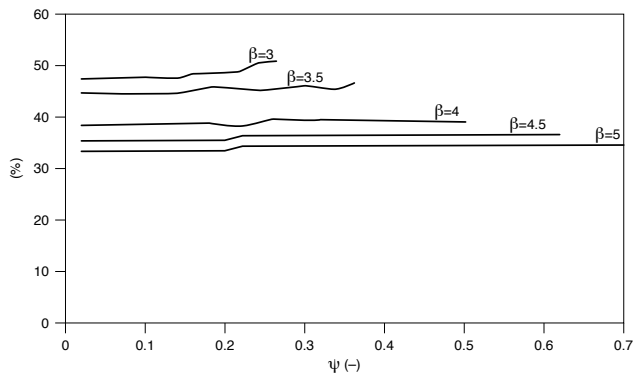


Figure 11  
Collector exergy loss  $T_c = 1500 K$ ,  $k = 1$ ,  $T_0 = 300 K$ ,  $K_{tc} = 0.01$ ,  $K_{tr} = 0.5$ .

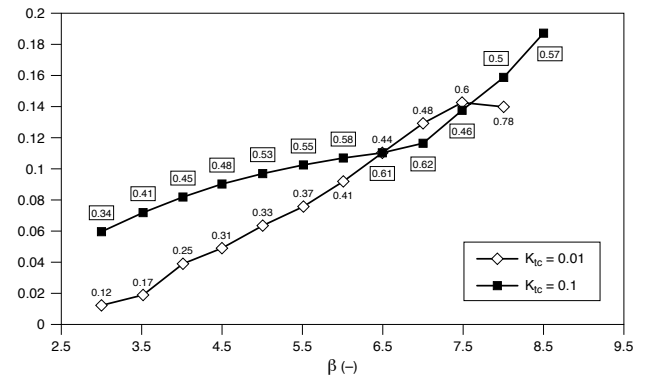


Figure 14  
Influence of  $K_{tc}$  on the optimum power of the engine  $T_c = 1500 K$ ,  $k = 1$ ,  $T_0 = 300 K$ ,  $\Delta T_0 = 5 K$ ,  $K_{tr} = 0.5$ .

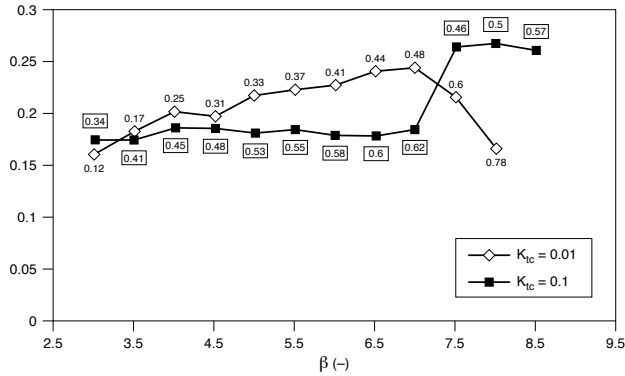


Figure 15

Influence of  $K_{tc}$  on the cycle efficiency  $T_c = 1500 K$ ,  $k = 1$ ,  $T_0 = 300 K$ ,  $\Delta T_0 = 5 K$ ,  $K_{tr} = 0.5$ .

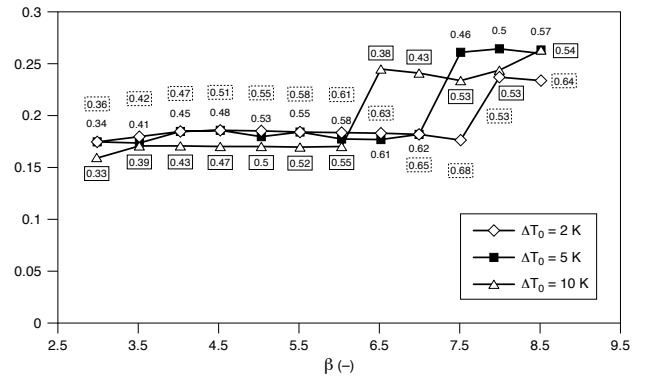


Figure 17

Influence of  $\Delta T_0$  on the cycle efficiency  $T_c = 1500 K$ ,  $K_{tc} = 0.1$ ,  $K_{tr} = 0.5$ ,  $k = 1$ ,  $T_0 = 300 K$ .

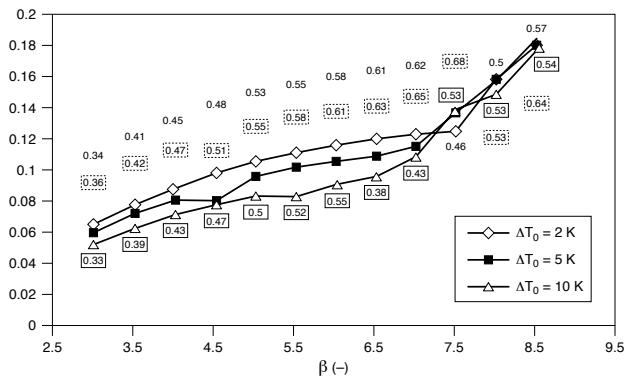


Figure 16

Influence of a  $\Delta T_0$  on the optimum power of the engine  $T_c = 1500 K$ ,  $K_{tc} = 0.1$ ,  $K_{tr} = 0.5$ ,  $k = 1$ ,  $T_0 = 300 K$ .

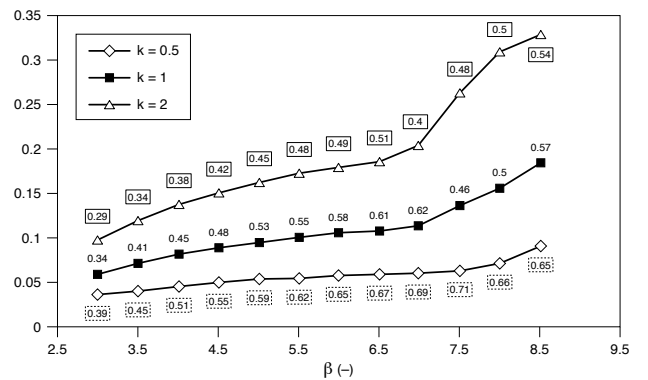


Figure 18

Influence of  $k$  on the optimum power of the engine  $T_c = 1500 K$ ,  $K_{tc} = 0.1$ ,  $K_{tr} = 0.5$ ,  $T_0 = 300 K$ .

increase of the power when  $\Delta T_0$  diminishes in the range of  $\beta$  (2-7); afterwhat the influence of  $\Delta T_0$  is not noticeable. The values corresponding to the optimum are quite similar, whatever is  $\Delta T_0$ . The Figure 17 reports the global efficiency of the system. It appears a poor influence until  $\beta$  equal 6, afterwhat we see a jump in the efficiency depending on the value of  $\Delta T_0$  ( $\Delta T_0 = 2 K$ ;  $\beta = 7.5$ ;  $\Delta T_0 = 5 K$ ;  $\beta = 6.75$ ;  $\Delta T_0 = 10 K$ ;  $\beta = 5.5$ ).

## 2.4 Influence of $k$

The  $k$  value increasing, the optimum power increases in a marked manner (Fig. 18), whereas the  $\psi$  corresponding to this optimum is slightly decreasing. The global efficiency of the system is not affected by  $k$ , until values of  $\beta$  near 6.5; for greater value of  $\beta$ , we see a similar influence of  $k$  (Fig. 19), as was seen for  $\Delta T_0$ .

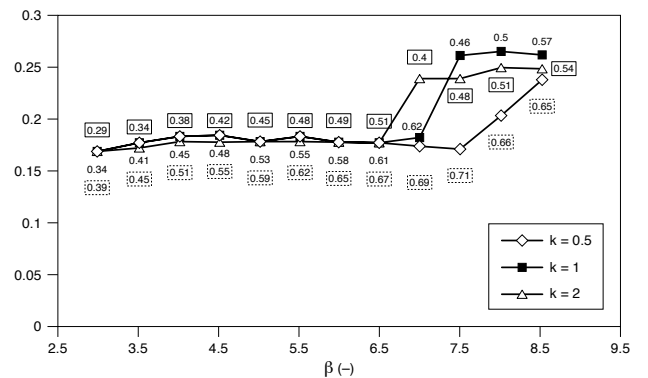


Figure 19

Influence of  $k$  on the cycle efficiency  $T_c = 1500 K$ ,  $K_{tc} = 0.1$ ,  $K_{tr} = 0.5$ ,  $T_0 = 300 K$ .

We renew here that  $k = \frac{K_T}{K_c}$ , so that it appears the interest to have higher value for the mass transfer conductance at the turbine, than at the compressor.

## CONCLUSIONS

Until now finite time thermodynamics (FFT) has been focused essentially on thermal irreversibility and the associated thermal gradients. It appears in this paper that mass transfer affects in the same way the optimum power of the system: it exists an optimum in term of  $\Psi$ , relative pressure loss of the system. This has been illustrated here, on a Brayton-Joule engine configuration.

A sensitivity analysis of this optimum power has been done. It results the interest to favour the mass transfer conductance of the compressor.

The pressure loss obtained at optimum is slightly higher than those used actually, and this value is an increasing value of  $\beta$  the pressure ratio of the system.

We notice also that the mass transfer at the optimum is related to the heat transfer, through for example  $\Delta T_0$ , and more significantly through  $K_{ic}$ . So it seems that the optimum power of an engine has to be determined taking into account both heat and mass transfer conductances, that are interdependant. Studies are developping now in that direction.

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